# Transformations Lesson #1:
## Functions and Graphs Review

**Warm-Up #1** *Introduction*

In this unit we will use graphs of functions learned in previous math courses, namely

| | |
|---|---|
| Polynomial Functions | Radical Functions |
| Absolute Value Functions | Rational Functions |

This lesson is a review of some of the properties of these functions.

### Polynomial Functions

A polynomial function in $x$ is a function in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0,$$

where

- $a_0, a_1, a_2, \dots a_n$ are real numbers, $a_n \neq 0$,
- $n \in W$.

$a_1, a_2, \dots a_n$ are called coefficients. $a_n$, is called the leading coefficient and $a_0$ is the constant term. The value of $n$ is the degree of the polynomial.

For example, the polynomial function $f(x) = 7x^3 + x^4 - 8x^2 + 5$ has

degree 4, leading coefficient 1, and constant term 5

Three common polynomial functions we will use for transformations are

- Linear Functions
- Quadratic Functions
- Cubic Functions

### Linear Functions

A linear function is a polynomial function of degree 1 of the form $f(x) = ax + b$.

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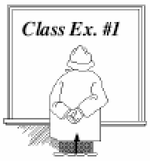
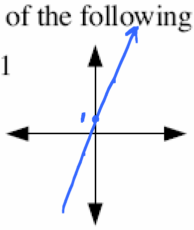


Sketch each of the following linear functions and answer the questions which follow.

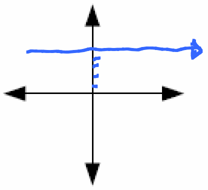
**i)** $y = 3x + 1$



**a)** What is the domain? $\{x \mid x \in R\}$

**b)** What is the range? $\{y \mid y \in R\}$

**ii)** $y = 5$



**a)** What is the domain? $\{x \mid x \in R\}$

**b)** What is the range? $\{y \mid y = 5\}$

**c)** What is another name for the function in ii)? Constant

## Quadratic Functions

A quadratic function is a polynomial function of degree 2 which can be written in general or standard form.

A quadratic function written in the form

$$f(x) = ax^2 + bx + c \text{ where } a \neq 0 \text{ is called the general form.}$$

A quadratic function or equation written in the form

$$f(x) = a(x - p)^2 + q \text{ where } a \neq 0 \text{ is called the standard form.}$$

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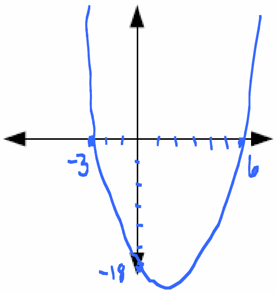
*Class Ex. #2*

**a)** Use a graphing calculator to sketch the graph of $y = x^2 - 3x - 18$ and determine:



**i)** the zeros $-3, 6$ **ii)** the $y$-intercept $-18$

**iii)** the coordinates of the vertex $V\left(\frac{3}{2}, -\frac{81}{4}\right)$

**iv)** the domain and range.

$D: \{x \mid x \in R\}$ $R: \{y \mid y \geq -\frac{81}{4}, y \in R\}$

**b)** Use factoring to determine the zeros.

$x^2 - 3x - 18 = (x-6)(x+3)$ $x - 6 = 0$, $x = 6$ $x + 3 = 0$, $x = -3$

**c)** Rewrite the quadratic function in standard form.
Explain how this form helps determine the coordinates of the vertex.

$\frac{1}{2} \cdot 3 = \frac{3}{2}$

$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$x^2 - 3x - 18$

$x^2 - 3x + \frac{9}{4} - \frac{9}{4} - 18$

$\left(x^2 - 3x + \frac{9}{4}\right) - \frac{9}{4} - \frac{72}{4}$

$\left(x - \frac{3}{2}\right)\left(x - \frac{3}{2}\right) - \frac{81}{4}$

$\left(x - \frac{3}{2}\right)^2 - \frac{81}{4}$

$V\left(\frac{3}{2}, -\frac{81}{4}\right)$

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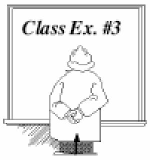
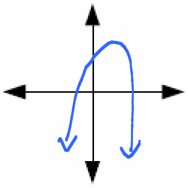

*Class Ex. #3*

Use a graphing calculator to sketch the graph of the quadratic function $f(x) = -3x^2 + 4x + 1$.



**a)** Use the features of a graphing calculator to determine the zeros to the nearest hundredth.

$-0.22$ , $1.55$

**b)** Use the quadratic formula to determine the exact values of the zeros in simplest radical form.

$a = -3$

$b = 4$

$c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-4 \pm \sqrt{28}}{-6}$ $= \frac{-2 \pm \sqrt{7}}{3}$

$x = \frac{-4 \pm \sqrt{4^2 - 4(-3)(1)}}{2(-3)}$ $= \frac{-4 \pm 2\sqrt{7}}{-6}$

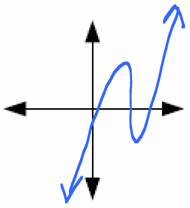
zeros $\frac{-2+\sqrt{7}}{3}$, $\frac{-2-\sqrt{7}}{3}$

## Cubic Functions

A cubic function is a polynomial function of degree 3 of the form $f(x) = ax^3 + bx^2 + cx + d,\ a \neq 0$.

**Class Ex. #4**

Use a graphing calculator to sketch the graph of the cubic function $y = x^3 - 8x^2 + 16x - 8$.



**a)** Determine the zeros to the nearest tenth. 0.8, 2.0, 5.2

**b)** Determine the exact value of the zeros.

possible factors ±1, ±2, ±4, ±8

$f(2) = 2^3 - 8(2)^2 + 16(2) - 8$

$= 0$

$$\begin{array}{r} x^2 - 6x + 4 \\ x-2\ \overline{)\ x^3 - 8x^2 + 16x - 8} \\ \underline{x^3 - 2x^2\phantom{+16x-8}} \\ -6x^2 + 16x\phantom{-8} \\ \underline{-6x^2 + 12x\phantom{-8}} \\ 4x - 8 \\ \underline{4x - 8} \\ 0 \end{array}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$$

zeros, 2, $3+\sqrt{5}$, $3-\sqrt{5}$

## Absolute Value Function

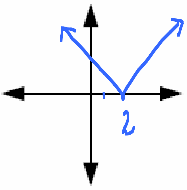
An absolute value function is a function of the form $f(x) = |x|$, where

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

**Class Ex. #5**

Sketch the graph of the absolute value function $y = |x - 2|$ and determine the domain and range.

domain $\{x \mid x \in R\}$ range $\{y \mid y \geq 0, y \in R\}$



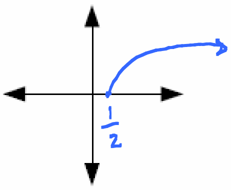
## Radical Functions

A radical function is a function which contains a variable in the radicand such as $f(x) = \sqrt{x}$.

**Class Ex. #6**

Sketch the radical function $y = \sqrt{2x - 1}$.
Determine the domain and range of the function.

domain $\{x \mid x \geq \frac{1}{2}, x \in R\}$ range $\{y \mid y \geq 0, y \in R\}$



## *Rational Functions*

Rational functions are functions of the form $f(x) = \frac{n(x)}{d(x)}$ where $n(x)$ and $d(x)$ are polynomials and $d(x) \neq 0$.

**Asymptote** - Some rational functions have an asymptote - an imaginary line to which the extreme ends of a graph approaches closer and closer as $|x|$ or $|y|$ increases.

Asymptotes are represented by dotted lines on the graph of a function. They are not part of the graph of a function, but help to show the nature of the function.

The dotted lines for an asymptote will not appear when the calculator is in dot mode or decimal connected mode. In the Greek language asymptote means "not meeting"

**Point of Discontinuity** - Others rational functions have a point of discontinuity - a break in the graph. The point of discontinuity can be seen using the Zoom Decimal window (or multiple of the Zoom Decimal window).

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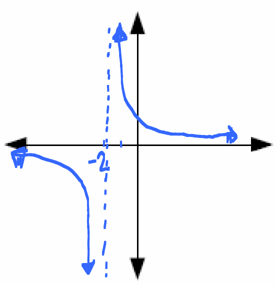
*Class Ex. #7*

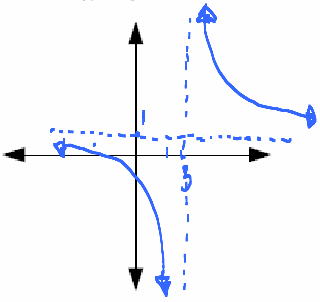
Use a graphing calculator to sketch the graph of each of the following rational functions. State the domain, the range, the equations of any asymptotes, and the coordinates of any points of discontinuity.

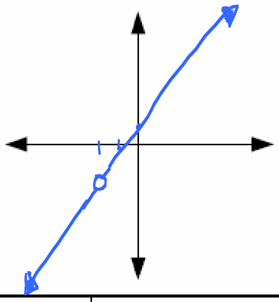
**a)** $y = \frac{1}{x+2}$ **b)** $y = \frac{x+2}{x-3}$ **c)** $y = \frac{x^2+3x+2}{x+2}$

$\frac{(x+2)(x+1)}{(x+2)}$

$y = (x+1)$ where $x \neq -2$







| | a) | b) | c) |
|---|---|---|---|
| Domain | $\{x \mid x \neq -2, x \in R\}$ | $\{x \mid x \neq 3, x \in R\}$ | $\{x \mid x \neq -2, x \in R\}$ |
| Range | $\{y \mid y \neq 0, y \in R\}$ | $\{y \mid y \neq 1, y \in R\}$ | $\{y \mid y \neq -1, y \in R\}$ |
| Equation of Vertical Asymptote | $x = -2$ | $x = 3$ | None |
| Equation of Horizontal Asymptote | $y = 0$ | $y = 1$ | None |
| Point of Discontinuity | None | None | $(-2, -1)$ |

**Complete Assignment Questions #1 - #11**

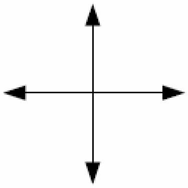
# Assignment

**1.** Graph each of the following functions and determine:

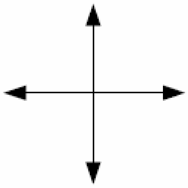
**i)** the zeros (to the nearest tenth if necessary)

**ii)** the domain and range

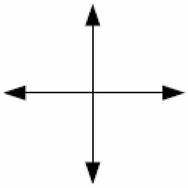
**a)** $y = -2x + 4$



**b)** $y = 5x - 10$



**c)** $y = x^2 - 2x - 15$

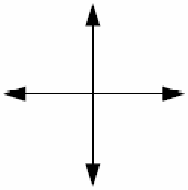


zeros

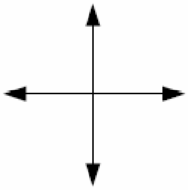
domain

range

**d)** $y = -x^2 - 3x - 4$



**e)** $y = (x + 1)(x + 2)(x - 5)$



zeros

domain

range

**2.** The linear function $y = ax + b$ is in slope $y$-intercept form.

**a)** Which parameter represents the slope of the line?

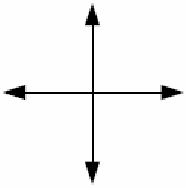
**b)** Which parameter represents the $y$-intercept?

**c)** If $a > 0$, describe the slope.

**d)** If $a < 0$, describe the slope.

**3.** How can you tell from the quadratic function $y = ax^2 + bx + c$ whether the graph of the function will open up or open down?

**4.** Use a graphing calculator to sketch the graph of the quadratic function $f(x) = 2x^2 - 5x - 6$.

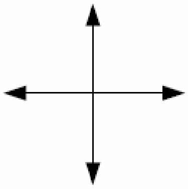


**a)** Use the features of a graphing calculator to determine the zeros to the nearest hundredth.

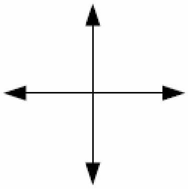
**b)** Use the quadratic formula to determine the exact values of the zeros in simplest radical form.

**5.** Sketch the graph of the following functions and state the domain and range.

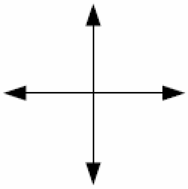
**a)** $y = |x| + 4$



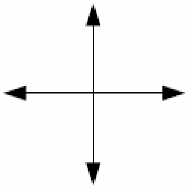
**b)** $y = |x + 3|$



**c)** $y = -|x + 2|$



**d)** $y = -|x| - 3$



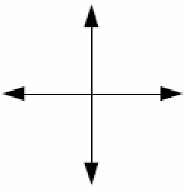
domain

range

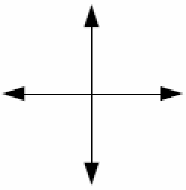
**6.** Which of the graphs in Question #5 open up? Open down?
What determines the direction of opening?

**7.** Sketch the graph of the following functions and state the domain and range.
What shape is each graph?

**a)** $y = \sqrt{16 - x^2}$



**b)** $y = -\sqrt{16 - x^2}$



domain

range

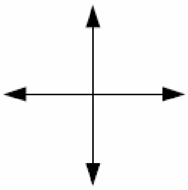
**8.** Combining the equations from Question #7a and #7b results in $y = \pm\sqrt{16 - x^2}$ .

**a)** Square both sides of the equation to rewrite in the form $x^2 + y^2 = k$

**b)** What geometrical shape is formed by combining the graphs from question #7a and #7b?

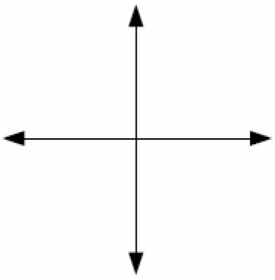
**c)** How does the value of $k$ relate to the radius of the circle?

**d)** Without using a graphing calculator, sketch the graph of $x^2 + y^2 = 49$.

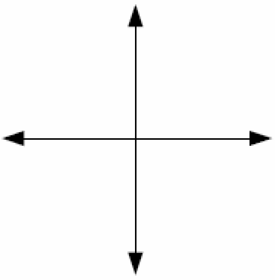


**9.** Use a graphing calculator to sketch the graph of each of the following rational functions. State the domain, the range, the equations of any asymptotes, and the coordinate of any points of discontinuity.

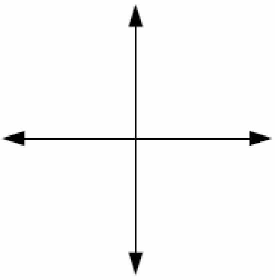
**a)** $y = \dfrac{2}{x - 4}$



**b)** $y = \dfrac{x}{x + 1}$



**c)** $y = \dfrac{x^2 - 9x + 20}{x - 5}$



| | a) | b) | c) |
|---|---|---|---|
| **Domain** | | | |
| **Range** | | | |
| **Equation of Vertical Asymptote** | | | |
| **Equation of Horizontal Asymptote** | | | |
| **Point of Discontinuity** | | | |

**Multiple Choice** **10.** The function $f$ is defined by $f(x) = 2x^2 + 3$. Which of the following represents $f(2x)$?

**A.** $2x^2 + 3$
**B.** $4x^2 + 3$
**C.** $8x^2 + 3$
**D.** $4x^2 + 6$

**Numerical Response** **11.** The rational expression $r(x) = \dfrac{x^2 - 8x + 12}{x - 6}$ has a point of discontinuity. The value of the $y$-coordinate of the point of discontinuity, to the nearest whole number, is _____ .

## *Answer Key*

**1. a)** zeros: 2
Domain: $\Re$
Range: $\Re$

**b)** zeros: 2
Domain: $\Re$
Range: $\Re$

**c)** zeros: –3, 5
Domain: $\Re$
Range: $\{y \mid y \geq -16, y \in \Re\}$

**d)** zeros: none
Domain: $\Re$
Range: $\left\{y \mid y \leq -\dfrac{7}{4}, y \in \Re\right\}$

**e)** zeros:–1, –2, 5
Domain: $\Re$
Range: $\Re$

**2. a)** a **b)** b **c)** rises from left to right **d)** falls from left to right

**3.** If $a > 0$, the graph opens up. If $a < 0$, the graph opens down.

**4. a)** –0.89, 3.39 **b)** $x = \dfrac{5 \pm \sqrt{73}}{4}$

**5. a)** Domain: $\Re$
Range: $\{y \mid y \geq 4, y \in \Re\}$

**b)** Domain: $\Re$
Range: $\{y \mid y \geq 0, y \in \Re\}$

**c)** Domain: $\Re$
Range: $\{y \mid y \leq 0, y \in \Re\}$

**d)** Domain: $\Re$
Range: $\{y \mid y \leq -3, y \in \Re\}$

**6.**
- Graphs a and b open up and graphs c and d open down.
- If there is a negative sign outside the absolute value symbol, the graph opens down. If there is no negative sign outside the absolute value symbol, the graph opens up.

**7. a)** Domain: $\{x \mid -4 \leq x \leq 4, x \in \Re\}$
Range: $\{y \mid 0 \leq y \leq 4, y \in \Re\}$

**b)** Domain: $\{x \mid -4 \leq x \leq 4, x \in \Re\}$
Range: $\{y \mid -4 \leq y \leq 0, y \in \Re\}$

**8. a)** $x^2 + y^2 = 16$ **b)** circle **c)** $k = r^2$ **d)** circle centre (0, 0) with radius 7

**9.**

| | a) | b) | c) |
|---|---|---|---|
| Domain | $\{x \mid x \neq 4, x \in \Re\}$ | $\{x \mid x \neq -1, x \in \Re\}$ | $\{x \mid x \neq 5, x \in \Re\}$ |
| Range | $\{y \mid y \neq 0, y \in \Re\}$ | $\{y \mid y \neq 1, y \in \Re\}$ | $\{y \mid y \neq 1, y \in \Re\}$ |
| Eqn. of Asymptote | | | |
| Vertical | $x = 4$ | $x = -1$ | none |
| Horizontal | $y = 0$ | $y = 1$ | none |
| Pt of Discontinuity | none | none | (5, 1) |

**10.** C **11.** 4