Transformations Lesson #2:
Function Notation and Inverse Functions

Function Notation

If $f(x) = x^2 + 4x + 5$, find the following in simplest form:

a) $f(2)$ $= 2^2+4(2)+5$ $4+8+5=17$

b) $2f(x)$ $= 2(x^2+4x+5)$ $= 2x^2+8x+10$

c) $f(2x)$ $= (2x)^2+4(2x)+5$ $4x^2+8x+5$

d) $f(x)+2$ $= x^2+4x+5+2$ $= x^2+4x+7$

e) $f(x-2)$ $= (x-2)^2+4(x-2)+5$ $(x-2)(x-2)+4x-8+5$ $x^2-2x-2x+4+4x-8+5$ x^2+1

f) $f(-x)$ $= (-x)^2+4(-x)+5$ $(-x)(-x)-4x+5$ x^2-4x+5

If $f(x) = |x|$, write the following in terms of the function f.

a) $|x| - 4$ $f(x)-4$

b) $|x-4|$ $f(x-4)$

c) $2|x|$ $2f(x)$

d) $|2x|$ $f(2x)$

e) $3 - |x|$ $3-f(x)$

f) $|x+3| - 2$ $f(x+3)-2$

g) $\frac{1}{|x|}$ $\frac{1}{f(x)}$

h) $|-x|$ $f(-x)$

If $f(x) = x^3$, write the following in terms of the function f.

a) $x^3 + 1$ $f(x)+1$

b) $(x+1)^3$ $f(x+1)$

c) $4x^3$ $4(fx)$

d) $(4x)^3$ $f(4x)$

e) $2x^3 + 3$ $2f(x)+3$

f) $2(x^3+3)$ $2[f(x)+3]$

g) $-x^3 + 2$ $-f(x)+2$

h) $(-x+4)^3$ $f(-x+4)$

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Inverse of a Function

A function relates elements of the domain to elements of the range. An inverse function reverses this process, i.e. the domain of an inverse function is the range of the original function and the range of the inverse function is the domain of the original function.

- If a function is defined by a set of ordered pairs, the inverse function is defined by interchanging the x and y the coordinates.
- If a function is defined by an equation, the inverse can be determined using the following procedure: Step 1: interchange x and y
  Step 2: solve for y
- If a function is defined graphically, the inverse can be determined by a reflection in the line $y = x$
- The inverse of the function $y = f(x)$ has the notation $y = f^{-1}(x)$, which is equivalent to $x = f(y)$.

a) Find the inverse of the following set of ordered pairs.

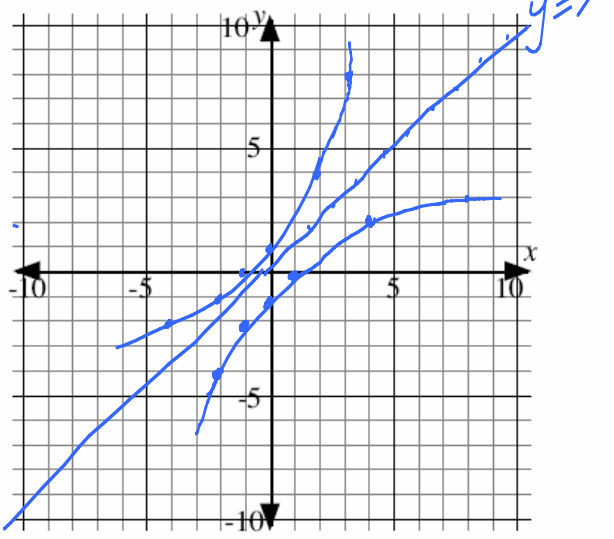
$\{(-4,-2), (-2,-1), (-1,0), (0,1), (2,4), (3,8)\}$

$\{(-2,-4), (-1,-2), (0,-1), (1,0), (4,2), (8,3)\}$

b) Graph the original set of ordered pairs and the inverse. Join each set of ordered pairs with a smooth curve.

c) Sketch the line of reflection. What is the equation of this line?

$y = x$



a) Find $f^{-1}(x)$ if $f(x) = 3x + 2$.

y

$x = 3y + 2$

$-2 \quad -2$

$\frac{x-2}{3} = \frac{3y}{3}$ $\quad y = \frac{x-2}{3}$

b) If $f(x) = x^2 - 1$, find $x = f(y)$ and solve for y.

$x = y^2 - 1$

$+1 \quad +1$

$\sqrt{x+1} = \sqrt{y^2}$

$y = \pm\sqrt{x+1}$

Complete Assignment Questions #1 - #6

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Assignment

1. If $f(x) = x^3 - 5$, find the following in simplest form:

a) $f(-1)$ **b)** $4f(x)$ **c)** $f(4x)$

d) $f(x) + 5$ **e)** $f(x + 5)$ **f)** $f(-x)$

2. If $f(x) = \sqrt{x}$, write the following in terms of the function f.

a) $\sqrt{x} - 1$ **b)** $\sqrt{x+3}$ **c)** $\sqrt{2x-1}$ **d)** $-3\sqrt{x}$

3. If $f(x) = x^2$, write the following in terms of the function f.

a) $x^2 + 3$ **b)** $(x + 3)^2$ **c)** $3x^2$

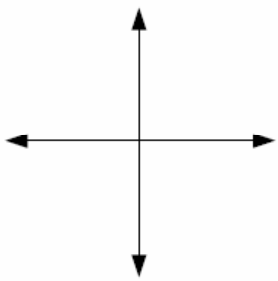
d) $(3x)^2$ **e)** $4x^2 - 7$ **f)** $4(x^2 - 7)$

g) $-2x^2 - 1$ **h)** $(-x + 4)^2$ **i)** $-3(-x - 2)^2$

4. **a)** Graph the function $f(x) = x^2 + 4$.

b) Graph the inverse of $f(x)$.

c) Find the equation of the inverse function in the form $x = f(y)$ and solve for y.



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Multiple Choice

5. A function f is defined by $y = 2x + 1$. Which of the following is the equation of the inverse of f?

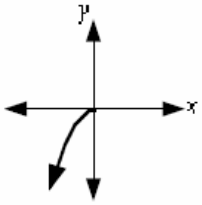
A. $x = \dfrac{y - 1}{2}$

B. $x = 2y + 1$

C. $y = \dfrac{x + 1}{2}$

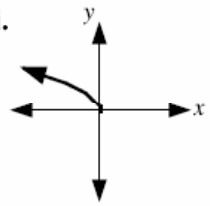
D. $y = 2x - 1$

6. The graph of the function $y = f(x)$ is shown in the diagram below.



Which of the following represents $f^{-1}(x)$?

i.



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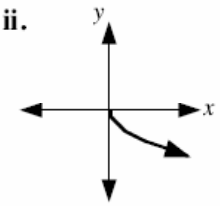

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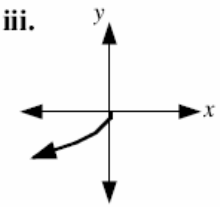

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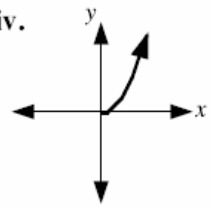

A. i
B. ii
C. iii
D. iv

Answer Key

1. **a)** -6 **b)** $4x^3 - 20$ **c)** $64x^3 - 5$ **d)** x^3 **e)** $x^3 + 15x^2 + 75x + 120$ **f)** $-x^3 - 5$

2. **a)** $f(x) - 1$ **b)** $f(x + 3)$ **c)** $f(2x - 1)$ **d)** $-3f(x)$

3. **a)** $f(x) + 3$ **b)** $f(x + 3)$ **c)** $3f(x)$ **d)** $f(3x)$ **e)** $4f(x) - 7$
f) $4(f(x) - 7)$ **g)** $-2f(x) - 1$ **h)** $f(-x + 4)$ **i)** $-3f(-x - 2)$

4. **c)** $x = y^2 + 4,\ y = \pm\sqrt{x - 4}$ **5.** B **6.** C

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