

# Transformations Lesson #3: Horizontal and Vertical Translations Part 1

## Transformations

A **transformation** is an operation which moves (or maps) a figure from an original position to a new position. Transformations we will consider are translations, reflections, expansions and compressions, reciprocal transformations, and absolute value transformations.

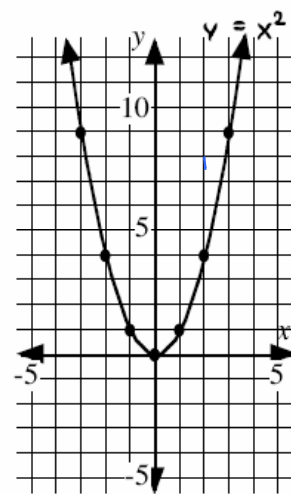
### Warm-Up #1

 Comparing the Graphs of  $y = f(x)$  and  $y - k = f(x)$  [or  $y = f(x) + k$ ]

#### Part 1

a) Complete the table of values. The first one has been completed.

$y = x^2$		$y - 3 = x^2$		$y + 3 = x^2$	
$x$	$y$	$x$	$y$	$x$	$y$
-3	9	-3	12	-3	6
-2	4	-2	7	-2	1
-1	1	-1	4	-1	-2
0	0	0	3	0	-3
1	1	1	4	1	-2
2	4	2	7	2	1
3	9	3	12	3	6



- b) Use the table of values in a) to graph and label each of the functions on the grid.
- c) What is the effect of the **parameter**  $k$  on the graph of the parabola  $y - k = x^2$ ?

A vertical shift of  $k$  units

#### Part 2

- a) Use a graphing calculator to graph the following functions:
- i)  $y = |x|$       ii)  $y = |x| + 2$       iii)  $y = |x| - 3$
- b) What is the effect of the **parameter**  $k$  on the graph of  $y = |x| + k$ ?

A vertical shift of  $k$  units

- c) What is the effect of the **parameter**  $k$  on the graph of the function  $y = f(x) + k$ ?

A vertical shift of  $k$  units

- d) What is the effect of the **parameter**  $k$  on the graph of the function  $y - k = f(x)$ ?

A vertical shift of  $k$  units

- e) Compared to the graph of  $y = f(x)$ , the graph of  $y - k = f(x)$  results in a Vertical shift of  $k$  units. If  $k > 0$ , the graph moves up. If  $k < 0$ , the graph moves down.



The notation  $y - k = f(x)$  is often used instead of  $y = f(x) + k$  to emphasize that this is a transformation on  $y$ . The concept of replacing  $y$  by  $y - k$  will be very important in this course.

**Warm-Up #2** Comparing the Graphs of  $y = f(x)$  and  $y = f(x - h)$

**Part 1**

a) Complete the table of values. The first one has been completed.

i)  $y = x^2$

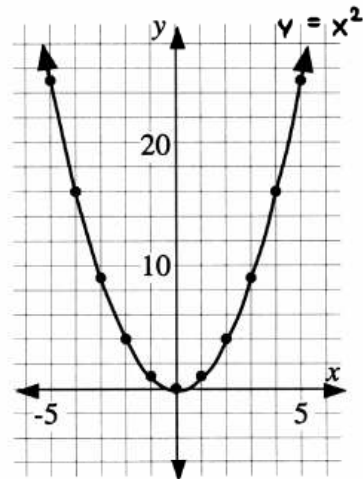
x	y
-4	16
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16

ii)  $y = (x - 3)^2$

x	y
-1	16
0	9
1	4
2	1
3	0
4	1
5	4
6	9
7	16

iii)  $y = (x + 3)^2$

x	y
-7	16
-6	9
-5	4
-4	1
-3	0
-2	1
-1	4
0	9
1	16



- b) Use the table of values in a) to graph and label each of the functions on the same grid.  
 c) What is the effect of the **parameter**  $h$  on the graph of the parabola  $y = (x - h)^2$ ?

horizontal shift, to the right if  $h > 0$   
 to the left if  $h < 0$

**Part 2**

a) Use a graphing calculator to graph the following functions:

- i)  $y = \sqrt{x}$       ii)  $y = \sqrt{x - 4}$       iii)  $y = \sqrt{x + 2}$

b) What is the effect of the **parameter**  $h$  on the graph of  $y = \sqrt{x - h}$ ?

hor. shift, to the right if  $h > 0$   
 to the left if  $h < 0$

c) What is the effect of the **parameter**  $h$  on the graph of the function  $y = f(x - h)$ ?

horizontal shift of  $h$  units

d) Compared to the graph of  $y = f(x)$ , the graph of  $y = f(x - h)$  results in a horizontal shift of  $h$  units. If  $h > 0$ , the graph moves right. If  $h < 0$  the graph moves left.

**Translations**

A **translation** is a transformation which slides each point of a figure the same distance in the same direction.



Given the function  $y = f(x)$ :

- replacing  $y$  with  $y - k$ , (i.e.  $y \rightarrow y - k$ ) describes a vertical translation.  
 $y - k = f(x)$  or  $y = f(x) + k$  describes a vertical translation.

- replacing  $x$  with  $x - h$ , (i.e.  $x \rightarrow x - h$ ) describes a horizontal translation.  
 $y = f(x - h)$  describes a horizontal translation.

In general, if

$$y - k = f(x - h)$$

or

$$y = f(x - h) + k \text{ then}$$

$k > 0$  the graph moves up  $\uparrow$   
 $k < 0$  the graph moves down  $\downarrow$   
 $h > 0$  the graph moves right  $\rightarrow$   
 $h < 0$  the graph moves left  $\leftarrow$



- If the graph of  $y = f(x)$  is transformed to the graph of  $y - 2 = f(x - 3)$ , the replacements for  $x$  and  $y$  are  $x \rightarrow x - 3$  and  $y \rightarrow y - 2$ . Under this transformation, all points on the graph of  $y = f(x)$  will move 3 units to the right and 2 units up. The point with coordinates (4, 6) will be translated to the point (7, 8). In general the point with coordinates (x, y) is translated to the point (x + 3, y + 2). This translation can be represented in mapping notation by  $(x, y) \rightarrow (x + 3, y + 2)$ .
- Do not confuse mapping notation with the notation we have used for replacements.



Describe how the graphs of the following functions relate to the graph of  $y = f(x)$ .

a)  $y = f(x - 3)$

hor translation  
3 units right

b)  $y = f(x) + 4$

vert translation  
4 units up

c)  $y - 1 = f(x + 10)$

$y = f(x + 10) + 1$   
 vert trans 1 unit up  
 hor trans 10 units left



The point (2, -3) lies on the graph of  $y = f(x)$ . State the coordinates of the image of this point under the following transformations.

a)  $y + 8 = f(x)$   $y = f(x) - 8$  vert trans 8 unit down  $(2, -3 - 8) \Rightarrow (2, -11)$

b)  $y = f(x - 7) + 5$  hor trans 7 right  $(2 + 7, -3 + 5) \Rightarrow (9, 2)$   
 vert trans 5 up

c)  $(x, y) \rightarrow (x + 2, y + 3)$  increase x by 2  $(2 + 2, -3 + 3) \Rightarrow (4, 0)$   
 increase y by 3



Class Ex. #3

Write the equation of the image of  $y = f(x)$  after each transformation.

a) A horizontal translation of 5 units left.

$$y = f(x+5)$$

b) A translation of 3 units up.

$$y = f(x) + 3 \text{ or } y - 3 = f(x)$$

c) A translation of  $m$  units right and  $p$  units down. d)  $(x, y) \rightarrow (x - 6, y + 1)$ .

$$y = f(x-m) - p$$

$$y = f(x+6) + 1$$

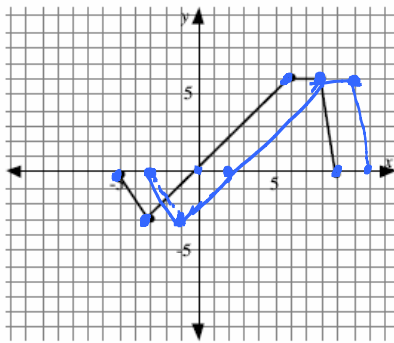
decrease  $x$  by 6  
increase  $y$  by 1



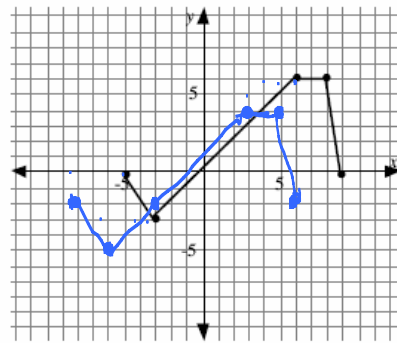
Class Ex. #4

The graph of  $f(x)$  is shown. Sketch;

a)  $y = f(x - 2)$  *2 units right*



b)  $y + 2 = f(x + 3)$  *3 unit left 2 unit down*



Class Ex. #5

What happens to the graph of the function  $y = f(x)$  if the following changes are made to its equation?

a) replace  $x$  with  $x + 2$   $y = f(x+2)$  *hor tran 2 left*

b) replace  $y$  with  $y - 8$   $y - 8 = f(x)$  *vert trans 8 up*  
 $y = f(x) + 8$

Complete Assignment Questions #1 - #9

## Assignment

1. Describe how the graphs of the following functions relate to the graph of  $y = f(x)$ .

a)  $y = f(x + 9)$

b)  $y = f(x) + 7$

c)  $y = f(x - 4) + 4$

d)  $y - 6 = f(x)$

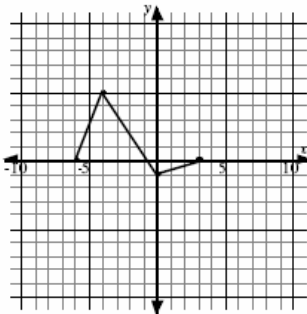
e)  $y = 3 + f(x - 5)$

f)  $y + 2 = f(x + 3) - 10$

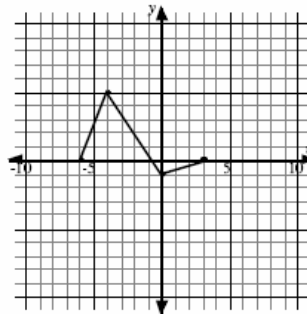
2. Write the equation of the image of  $y = f(x)$  after each transformation.
- A vertical translation of 10 units down.
  - A horizontal translation of 8 units right and a vertical translation of 9 units up.
  - A translation of  $t$  units up and  $s$  units left.
3. The function  $y = f(x)$  is transformed to  $y = f(x - h) + k$ . Find the values of  $h$  and  $k$  for the following translations.
- 7 units right      b) 4 units up and 2 units left      c)  $a$  units right and  $b$  units down.
4. The point  $(-3, 5)$  lies on the graph of  $y = f(x)$ . State the coordinates of the image of this point under the following transformations.
- $y = f(x) + 3$                       b)  $y + 5 = f(x + 2)$                       c)  $(x, y) \rightarrow (x - 7, y - 1)$

5. Given the graph of the function  $y = f(x)$  sketch the graph of the indicated function.

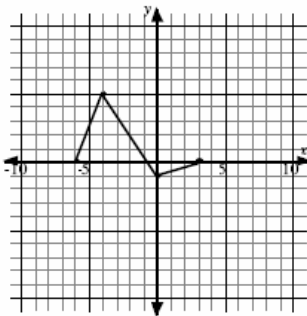
a)  $y = f(x - 4)$



b)  $y - 3 = f(x)$



c)  $y = f(x + 2) - 3$



d)  $y + 2 = f(x - 5)$

