

## Transformations Lesson #4: Horizontal and Vertical Translations Part 2

### Warm-Up

In the previous lesson we had the following note:

Given the function  $y = f(x)$ :

- replacing  $y$  with  $y - k$ , (i.e.  $y \rightarrow y - k$ ) describes a vertical translation.  
 $y - k = f(x)$  or  $y = f(x) + k$  describes a vertical translation.
- replacing  $x$  with  $x - h$ , (i.e.  $x \rightarrow x - h$ ) describes a horizontal translation.  
 $y = f(x - h)$  describes a horizontal translation.

In general, if  $y - k = f(x - h)$  or  $y = f(x - h) + k$  then



$k > 0$  the graph moves up  $\uparrow$   
 $k < 0$  the graph moves down  $\downarrow$   
 $h > 0$  the graph moves right  $\rightarrow$   
 $h < 0$  the graph moves left  $\leftarrow$



Class Ex. #1

Describe how the graph of the second function compares to the graph of the first function.

- |   |  |
|---|--|
| <p>a) <math>y = x^4</math>      <i>vert trans</i><br/> <math>y = x^4 + 3</math>      <i>3 units up</i></p>  | <p>b) <math>y = 6x - 3</math>      <i>hor trans</i><br/> <math>y = 6(x - 1) - 3</math>      <i>1 unit right</i></p>                    |
| <p>c) <math>y =  x </math>      <i>hor trans</i><br/>                                  <i>6 unit right</i><br/> <math>y =  x - 6  + 2</math>      <i>vert trans</i><br/>            <i>2 units up</i></p> | <p>d) <math>y = \frac{1}{\sqrt{x}}</math>      <i>hor trans</i><br/> <math>y = \frac{1}{\sqrt{x+1}}</math>      <i>1 unit left</i></p> |



Class Ex. #2

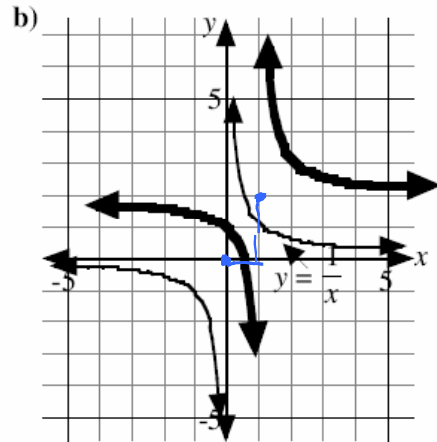
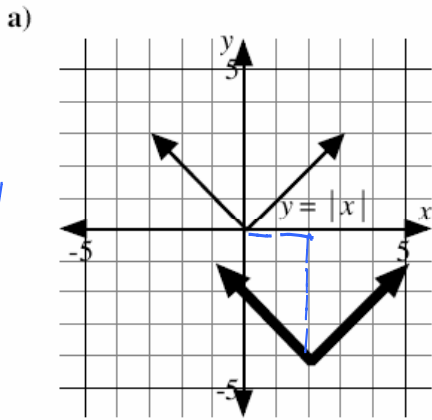
Write the equation of the image of:

- a)  $y = x^2$  after a horizontal translation of 3 units to the right.       $y = (x-3)^2$
- b)  $y = 10^x$  after a vertical translation of 2 units up.       $y = 10^x + 2$
- c)  $y = \sqrt{x}$  after a horizontal translation of 4 units to the left and a vertical translation of 3 units down.       $y = \sqrt{x+4} - 3$



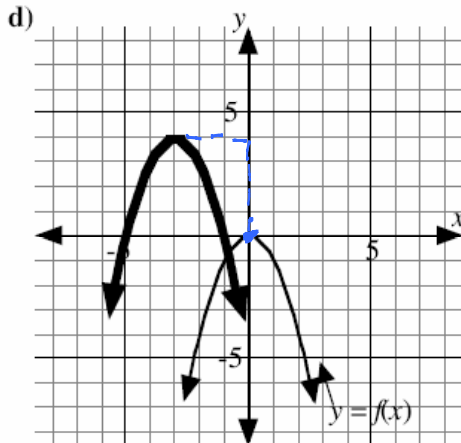
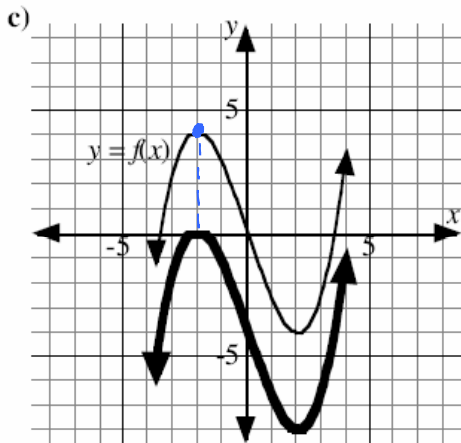
The function represented by the thick line is a transformation of the function represented by the thin line. Write an equation for each function represented by the thick line.

$y = |x-2| - 4$



$y = \frac{1}{x-1} + 2$

$y = f(x) - 4$



$y = f(x+3) + 4$



$y = \sqrt{x}$  is a radical function.

a) What vertical translation would be applied to  $y = \sqrt{x}$  so that the translation image passes through (16, 7)?

$(x, y)$        $y = \sqrt{x} + K$        $7 = \sqrt{16} + K$        $K = 3$   
 $7 = 4 + K$        $K = 3$   
 $-4 -4$       **vert tran 3 up**

b) What horizontal translation would be applied to  $y = \sqrt{x}$  so that the translation image passes through (17, 8)?

$x, y$        $y = \sqrt{x-h}$        $h = -47$   
 $8 = \sqrt{17-h}$       **hor tran 47 units left**  
 $64 = 17-h$   
 $h = 17-64$



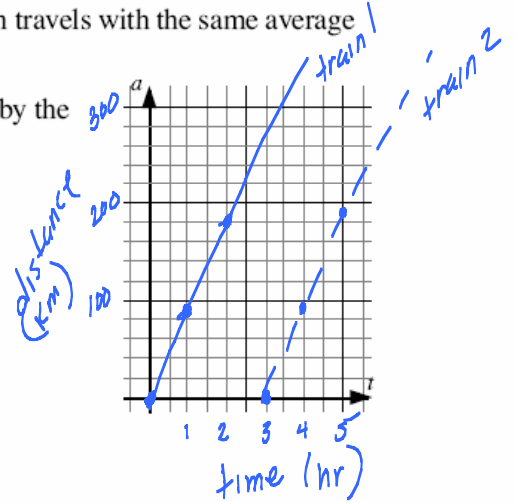
On a certain route trains travel at an average speed of 90km/h. The distance  $d$ , in kilometres, they travel can be described as a function of time,  $t$ , in hours, and represented by the equation  $d = f(t) = 90t$ .

A train leaves the station at 12:00 p.m. ( $t = 0$ ). A second train travels with the same average speed, but leaves 3 hours later.

- a) Write an equation which describes the distance travelled by the second train.

$$f(t) = 90(t-3)$$

- b) Sketch a distance time graph for each train on the grid.



Complete Assignment Questions #1 - #8

### Assignment

1. Describe how the graph of the second function compares to the graph of the first function.

a)  $y = x^3$   
 $y = x^3 - 1$

b)  $y = 7x - 1$   
 $y = 7(x - 3) - 1$

c)  $y = \cos x^\circ$   
 $y = \cos (x + 45)^\circ$

d)  $y = |x|$   
 $y + 3 = |x + 6|$

e)  $y = \frac{1}{x^2 + 1}$   
 $y - 2 = \frac{1}{(x - 3)^2 + 1}$

f)  $y = a^x$   
 $y = a^{x+1} + 1$

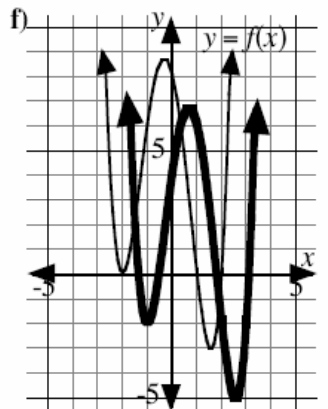
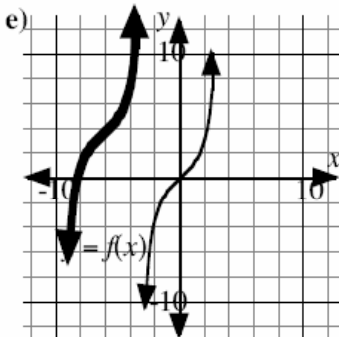
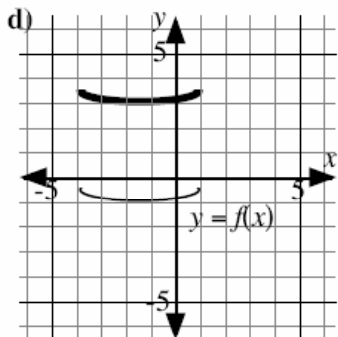
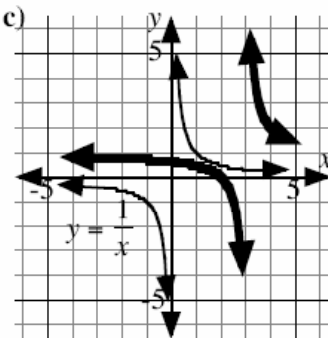
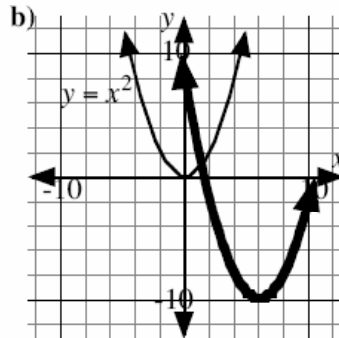
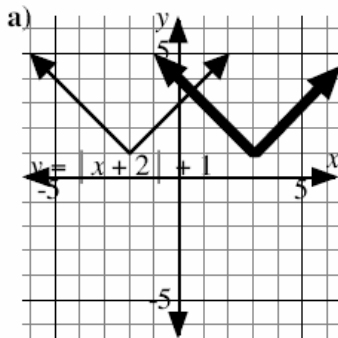
2. Write the equation of the image of:

a)  $y = x^4$  after a horizontal translation of 2 units to the left.

b)  $y = 2|x|$  after a translation of 3 units down and 1 unit left.

c)  $y = \frac{1}{\sqrt{x}}$  after a horizontal translation of 3 units to the right and a vertical translation of 2 units up.

3. The function represented by the thick line is a transformation of the function represented by the thin line. Write an equation for each function represented by the thick line.



4. a) What vertical translation would be applied to  $y = x^2$  so that the translation image passes through  $(3, 5)$ ?
- b) What horizontal translation would be applied to  $y = x^3 + 1$  so that the translation image passes through  $(5, 28)$ ?
- c) What horizontal translation would be applied to  $y = \frac{1}{x - 3}$  so that the translation image passes through  $(1, \frac{1}{2})$ ?

5. On a certain route into town, shuttle buses depart every 15 minutes from 06:30 until 07:30. The distance  $d$ , in kilometres, they travel can be described as a function of time,  $t$ , in hours, and represented by the equation  $d = f(t) = 60t$ .

If  $t = 0$  at 06:30, write an equation which represents the distance travelled by:

- a) the second bus                      b) the third bus                      c) the last bus

**Multiple Choice**

6. The graph of the function  $y = f(x)$  passes through the point  $(4, 7)$ . Under a transformation, the point  $(4, 7)$  is transformed to  $(6, 6)$ . A possible equation for the transformed function is

- A.  $y - 1 = f(x + 2)$   
 B.  $y - 2 = f(x + 1)$   
 C.  $y + 1 = f(x - 2)$   
 D.  $y + 2 = f(x - 1)$

**Numerical Response**

7. The function  $f(x) = \sqrt{x} + 5$  is transformed by a translation of 2 units down and 4 units to the left. The transformed function passes through the point  $(20, y)$ . To the nearest tenth, the value of  $y$  is \_\_\_\_\_.

8. The function  $r(x) = \frac{1}{x+3}$  is transformed by a translation of 3 units up and 5 units to the right. The transformed function passes through the point  $(x, 7)$ . The value of  $x$  to the nearest hundredth is \_\_\_\_\_.

**Answer Key**

1. a) vertical translation 1 unit down      b) translation 3 units right  
 c) horizontal translation 45° left      d) translation 6 units left and 3 units down  
 e) translation 3 units right and 2 units up      f) translation 1 unit left and 1 unit up
2. a)  $y = (x + 2)^4$     b)  $y = 2|x + 1| - 3$     c)  $y = \frac{1}{\sqrt{x - 3}} + 2$
3. a)  $y = |x - 3| + 1$       b)  $y = (x - 6)^2 - 10$       c)  $y = \frac{1}{x - 3} + 1$   
 d)  $y = f(x) + 4$       e)  $y = f(x + 6) + 4$       f)  $y = f(x - 1) - 2$
4. a) vertical translation 4 units down      b) horizontal translation 2 units right  
 c) horizontal translation 4 units left
5. a)  $d = 60\left(t - \frac{1}{4}\right)$       b)  $d = 60\left(t - \frac{1}{2}\right)$       c)  $d = 60(t - 1)$
6. C      7. 7.9      8. 2.25

