

Transformations Lesson #5: Reflections Part 1

Invariant Points

Invariant points are points on a graph which do not move after a transformation.

Warm-Up #1

Comparing the Graphs of $y = f(x)$ and $y = -f(x)$

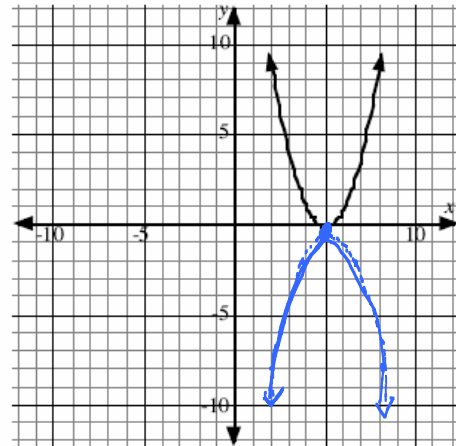
Part 1

- a) The graph of $y = f(x) = x^2 - 10x + 25$ is shown.
Write an equation which represents $y = -f(x)$.

$$y = -(x^2 - 10x + 25) \\ = -x^2 + 10x - 25$$

- b) Use a graphing calculator to sketch $y = -f(x)$ and show the graph on the grid.
c) State the coordinates of the invariant point(s).

$$(5, 0)$$



Part 2

- a) The graph of $y = f(x) = x^3 - 8$ is shown.
Write an equation which represents $y = -f(x)$.

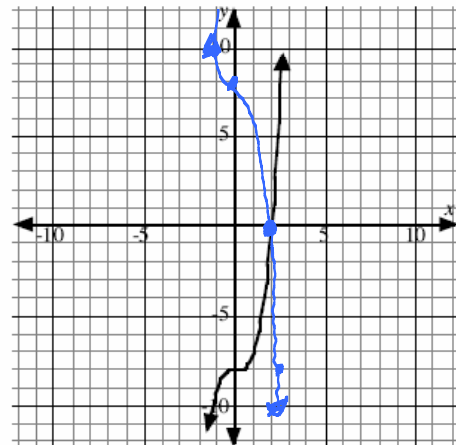
$$y = -(x^3 - 8) \quad y = -x^3 + 8$$

- b) Use a graphing calculator to sketch $y = -f(x)$ and show the graph on the grid.
c) State the coordinates of the invariant point(s).

$$(2, 0)$$

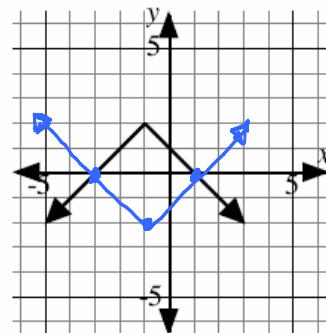
- d) How does the graph of $y = -f(x)$ compare with the graph of $y = f(x)$?

reflection in the x-axis



Part 3

The graph of $y = f(x)$ is shown.
Sketch the graph of $y = -f(x)$.



Warm-Up #2

Comparing the Graphs of $y = f(x)$ and $y = f(-x)$

invariant Calc #1 value or set x=0 solve for y yint 25-

Part 1

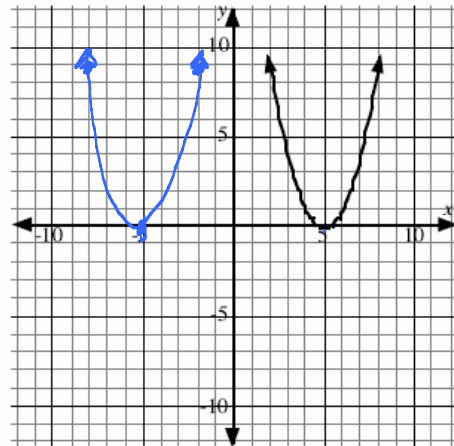
- a) The graph of $y = f(x) = x^2 - 10x + 25$ is shown. Write an equation which represents $y = f(-x)$.

$$y = (-x)^2 - 10(-x) + 25$$

$$y = x^2 + 10x + 25$$

- b) Use a graphing calculator to sketch $y = f(-x)$ and show the graph on the grid.
 c) State the coordinates of the invariant point(s).

$$(0, 25)$$



Part 2

- a) The graph of $y = f(x) = x^3 - 8$ is shown. Write an equation which represents $y = f(-x)$.

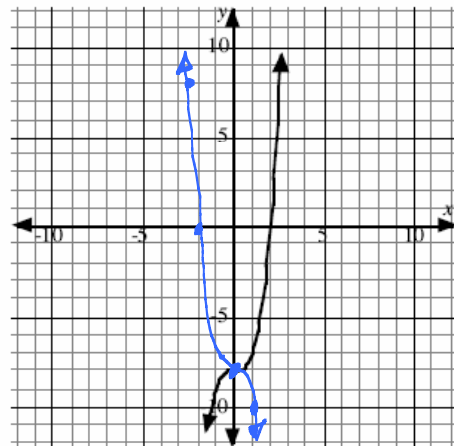
$$y = (-x)^3 - 8 \quad y = -x^3 - 8$$

- b) Use a graphing calculator to sketch $y = f(-x)$ and show the graph on the grid.
 c) State the coordinates of the invariant point(s).

$$(0, -8)$$

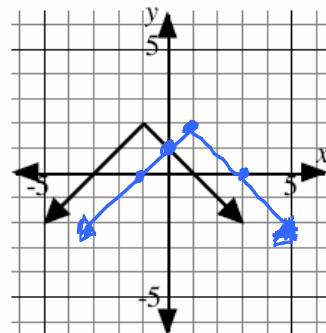
- d) How does the graph of $y = f(-x)$ compare with the graph of $y = f(x)$?

a reflection in the y axis



Part 3

The graph of $y = f(x)$ is shown. Sketch the graph of $y = f(-x)$.



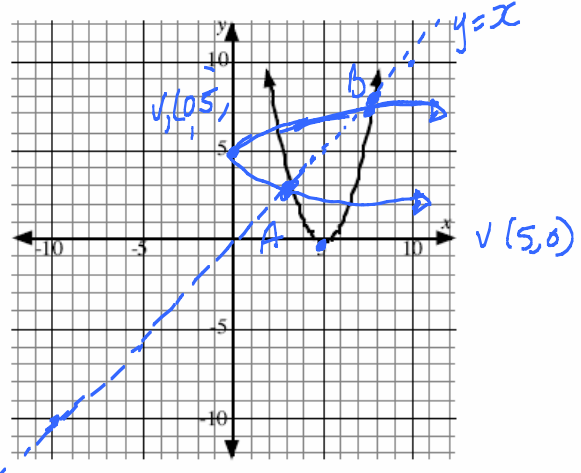
Warm-Up #3

Comparing the Graphs of $y = f(x)$ and $y = f^{-1}(x)$ or $x = f(y)$ *inverse*

Part 1

- a) The graph of $y = f(x) = (x - 5)^2$ is shown. Write an equation which represents $x = f(y)$ and solve for y .

$$\begin{aligned} \sqrt{x} &= \sqrt{(y-5)^2} \\ \pm\sqrt{x} + 5 &= y - 5 + 5 \\ y &= \pm\sqrt{x} + 5 \end{aligned}$$



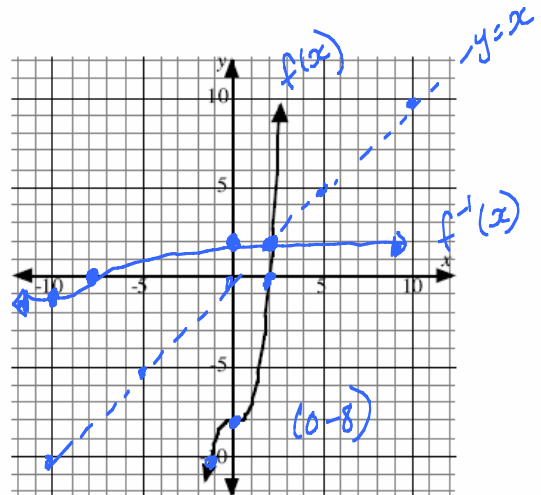
- b) Use a graphing calculator to sketch $x = f(y)$ and show the graph on the grid.
 c) Although there are four points of intersection of the graphs, explain why there are only two invariant points. Mark the invariant points on the grid.

p+A p+B

Part 2

- a) The graph of $y = f(x) = x^3 - 8$ is shown. Write an equation which represents $y = f^{-1}(x)$.

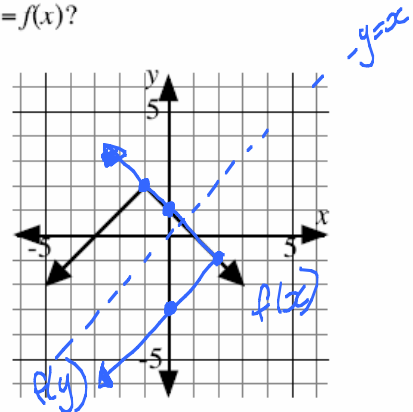
$$\begin{aligned} x + 8 &= y^3 - 8 + 8 \\ 3\sqrt{x+8} &= 3\sqrt{y^3} \\ y &= \sqrt[3]{x+8} \end{aligned}$$



- b) Use a graphing calculator to sketch $y = f^{-1}(x)$ and show the graph on the grid.
 c) Mark the invariant point(s) on the grid.
 d) How does the graph of $x = f(y)$ compare with the graph of $y = f(x)$?

Part 3

The graph of $y = f(x)$ is shown. Sketch the graph of $x = f(y)$.



Reflections

A **reflection** is a transformation which reflects (or flips) a figure about a line.

Fill in the following blanks which summarize Warm-Up #1 through to Warm-Up #3.



<i>Reflection</i>	<i>Function</i>	<i>Graph</i>
Reflection in the x -axis	If the graph of $y = f(x)$ is reflected in the <u>x-axis</u> , then it is the graph of <u>$y = -f(x)$</u> .	
Reflection in the y -axis	If the graph of $y = f(x)$ is reflected in the <u>y axis</u> , then it is the graph of <u>$y = f(-x)$</u> .	
Reflection in the line $y = x$	If the graph of $y = f(x)$ is reflected in the line <u>$y = x$</u> , then it is the graph of <u>$y = f^{-1}(x)$</u> or <u>$x = f(y)$</u> .	



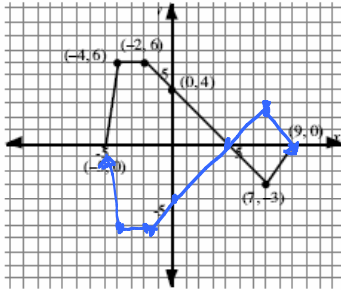
Note Given the function $y = f(x)$:

- replacing x with $-x$, (i.e. $x \rightarrow -x$) describes a reflection in the y -axis.
 $y = f(-x)$ describes a reflection in the y -axis.
- replacing y with $-y$, (i.e. $y \rightarrow -y$) describes a reflection in the x -axis.
 $-y = f(x)$ or $y = -f(x)$ describes a reflection in the x -axis.
- interchanging x and y , (i.e. $x \rightarrow y$, $y \rightarrow x$) describes a reflection in the line $y = x$
 $x = f(y)$ or $y = f^{-1}(x)$ describes a reflection in the line $y = x$.

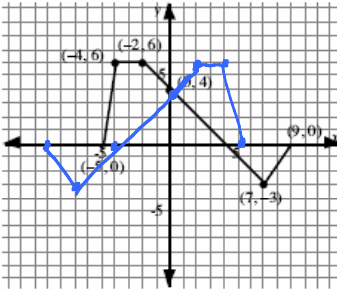


The graph of $y = f(x)$ is shown. Sketch:

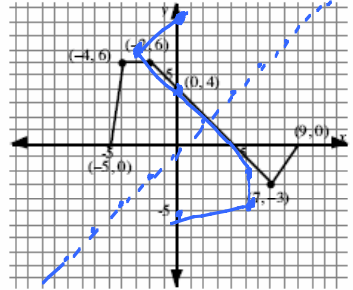
a) $y = -f(x)$



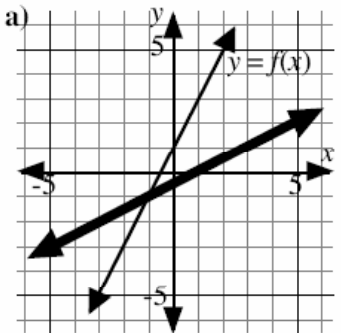
b) $y = f(-x)$



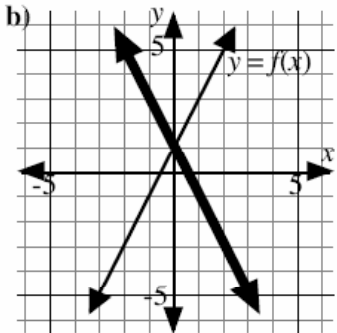
c) $x = f(y)$



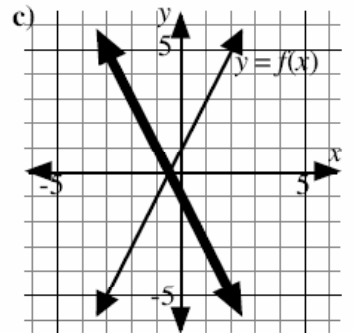
The graph drawn in the thick line is a reflection of the graph drawn in the thin line. Write an equation for each graph drawn in the thick line.



$y = f^{-1}(x)$



$y = f(-x)$

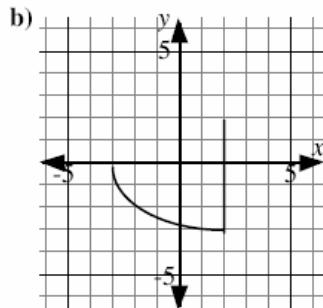
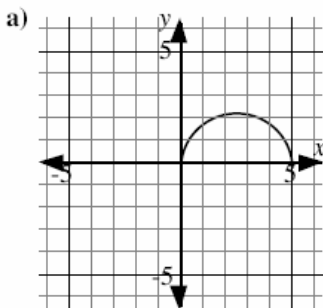


$y = -f(x)$

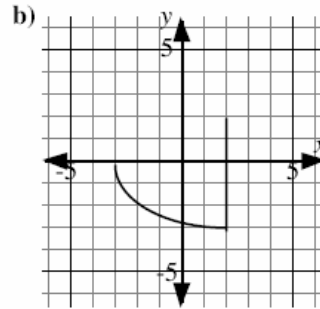
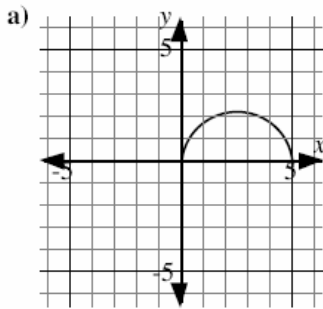
Complete Assignment Questions #1 - #7

Assignment

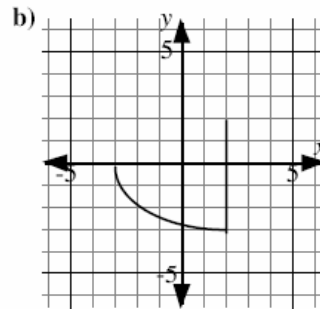
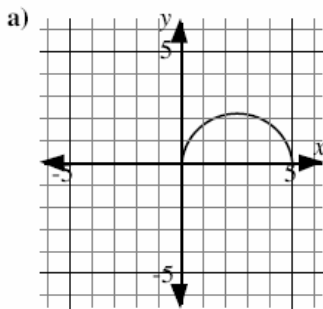
1. The graph of $y = f(x)$ is shown. Sketch the graph of $y = -f(x)$.



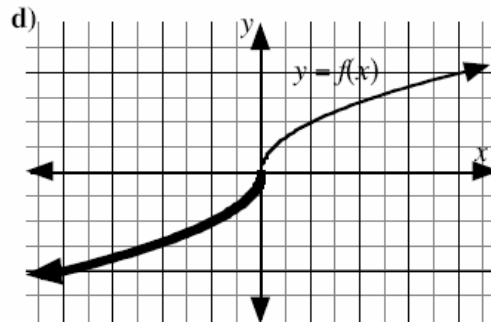
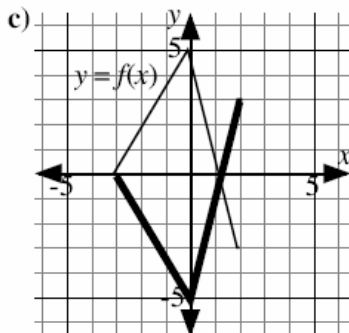
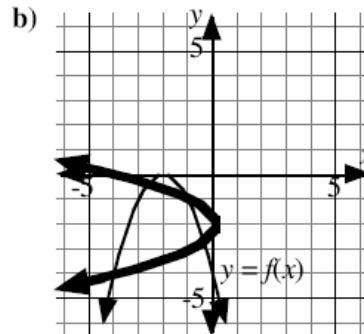
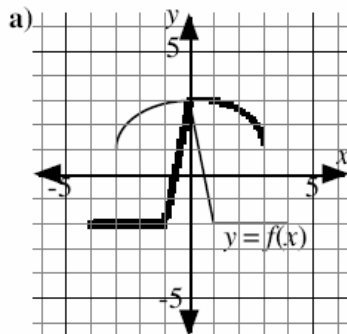
2. The graph of $y = f(x)$ is shown. Sketch the graph of $y = f(-x)$.



3. The graph of $y = f(x)$ is shown. Sketch the graph of $x = f(y)$.



4. The graph drawn in the thick line is a transformation of the graph drawn in the thin line. Write an equation for each graph drawn in the thick line and state whether this graph represents a function.



5. The function $y = f(x)$ is transformed to the function below. Given that there are invariant points, describe the location of these points.

- a) $y = -f(x)$ b) $y = f(-x)$ c) $x = f(y)$

6. The point (x, y) lies of the graph of the function $y = f(x)$. State the coordinates of the image of (x, y) under the following transformations:

- a) $y = -f(x)$ b) $y = f(-x)$ c) $x = f(y)$

Multiple Choice

7. Consider the graph of the function $f(x) = x^2$. Which of the following transformations would result in an identical graph?

- A. $-f(x)$
 B. $f(-x)$
 C. $-f(-x)$
 D. $f(x + 1)$

Answer Key

1. a) and b) graph is reflected in x-axis 2. a) and b) graph is reflected in y-axis
 3. a) and b) graph is reflected in the line $y = x$
 4. a) $y = f(-x)$ is a function b) $x = f(y)$ is a not a function
 c) $y = -f(x)$ is a function d) $y = -f(-x)$ is a function
 5. a) on the x-axis b) on the y-axis c) on the line $y = x$
 6. a) $(x, -y)$ b) $(-x, y)$ c) (y, x) 7. B

