Transformations Lesson #6:
Reflections Part 2

Warm-Up

In the previous lesson we had the following note:

Given the function $y = f(x)$:

- replacing x with $-x$, (*i.e.* $x \rightarrow -x$) describes a reflection in the y-axis.
  $y = f(-x)$ describes a reflection in the y-axis.

- replacing y with $-y$, (*i.e.* $y \rightarrow -y$) describes a reflection in the x-axis.
  $-y = f(x)$ or $y = -f(x)$ describes a reflection in the x-axis.

- interchanging x and y, (*i.e.* $x \rightarrow y$, $y \rightarrow x$) describes a reflection in the line $y = x$
  $x = f(y)$ or $y = f^{-1}(x)$ describes a reflection in the line $y = x$.

Class Ex. #1

Write the equation of the image of:
a) $y = x^2$ after a reflection in the line $y = x$

$\sqrt{x} = \sqrt{y^2} \qquad y = \pm\sqrt{x}$

b) $y = 10^x$ after a reflection in the y-axis

$y = 10^{-x}$

c) $y = \sqrt{x}$ after a reflection in the x-axis.

$y = -\sqrt{x}$

Class Ex. #2

Describe how the graph of the second function compares to the graph of the first function.

a) $y = x^3$
   $y = -x^3$ or $y = (-x)^3$

   reflection in the x or y axis

b) $y = 2^x$
 $x = 2^y$

   reflection in the $y = x$ axis

c) $y = \sin x$
   $y = \sin(-x)$

   reflection in the y axis

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Class Ex. #3

The graph drawn in the thick line is a transformation of the graph drawn in the thin line. Write an equation for each graph drawn in the thick line and state whether this graph represents a function.

a)

$y = x^2$

$\sqrt{x} = \sqrt{y^2}$

$y = \pm\sqrt{x}$
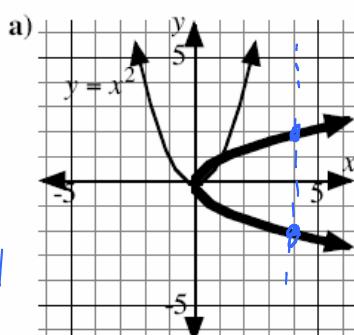
Fails Vertical Line Test

Not a Function   reflection in the $y = x$ axis

b)

$y = |x|$

reflection in the x-axis

$y = -|x|$
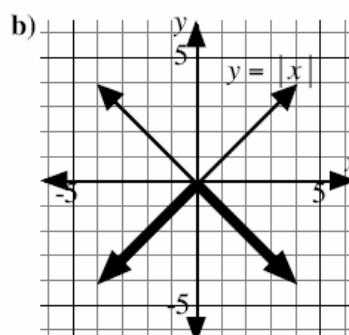
✗ **Vertical Line Test to show a one to one relationship for function**

Passes Vertical Line Test in a function

Class Ex. #4

a) Sketch the graph of $f(x) = \dfrac{6}{x^2 + 3}$.

b) Write the equation for

i) $y = -f(x)$ ii) $y = f(-x)$ iii) $x = f(y)$

$y = \dfrac{-6}{x^2 + 3}$

$y = \dfrac{6}{(-x)^2 + 3}$

$= \dfrac{6}{x^2 + 3}$

$x = \dfrac{6}{y^2 + 3}$

$x(y^2 + 3) = \dfrac{6}{x} \cdot x$

$y^2 + 3 = \dfrac{6}{x} - 3$

$\sqrt{y^2} = \sqrt{\dfrac{6}{x} - \dfrac{3x}{x}}$

$y = \pm \sqrt{\dfrac{6 - 3x}{x}}$

c) Sketch each graph in b) and state whether the graph represents a function.

i) $y = -f(x)$ ii) $y = f(-x)$ iii) $x = f(y)$

Class Ex. #5
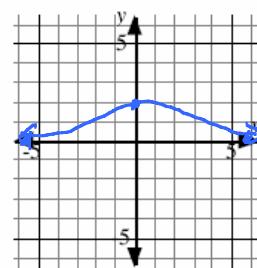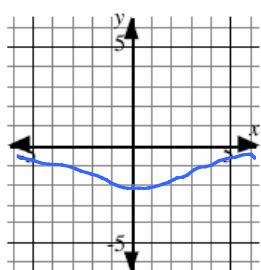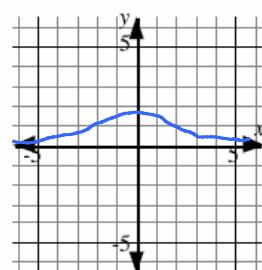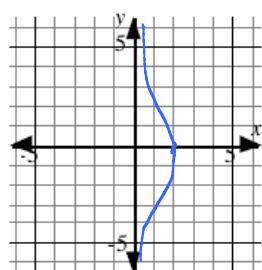
a) Given $f(x) = 3x + 2$, determine:

i) $f^{-1}(x)$ ii) $f^{-1}(f(x))$ iii) $f(f^{-1}(x))$

$x = 3y + 2$

$\dfrac{x - 2}{3} = \dfrac{3y}{3}$ $y = \dfrac{x-2}{3}$

$y = \dfrac{(3x + 2) - 2}{3} = \dfrac{3x}{3} = x$

$y = x$

$y = 3(x) + 2$

$y = 3\left(\dfrac{x-2}{3}\right) + 2$

$= x - 2 + 2$

$y = x$

b) The graph of $y = 3x + 2$ is given. Sketch each graph in a);

i) $y = f^{-1}(x)$ ii) $y = f^{-1}(f(x))$ iii) $y = f(f^{-1}(x))$
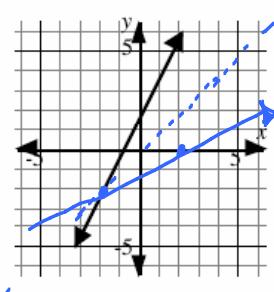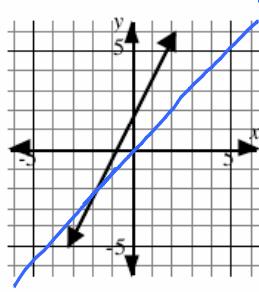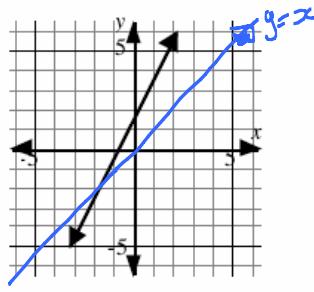
Note The graphs of $y = f^{-1}(f(x))$ and $y = f(f^{-1}(x))$ will always be the line with equation $y = x$ regardless of the function f.

Complete Assignment Questions #1 - #8

Assignment

1. Write the equation of the image of:

   a) $y = \dfrac{1}{x}$ after a reflection in the line $y = x$

   b) $y = x^3 + x$ after a reflection in the y-axis

   c) $y = |x|$ after a reflection in the x-axis.

   d) $y = \sqrt{x - 2}$ after a reflection in the line $y = x$

   e) $y = x^2 + 1$ after a reflection in the y-axis

   f) $y = \cos x$ after a reflection in the x-axis

2. Describe how the graph of the second function compares to the graph of the first function.

   a) $y = 3x + 1$
      $y = -3x - 1$

   b) $y = 3x + 1$
      $y = -3x + 1$

   c) $y = 3x + 1$
      $x = 3y + 1$

   d) $y = 10^x$
      $y = 10^{-x}$

   e) $y = 10^x$
      $y = -10^x$

   f) $y = 4x^2$
      $y = \pm\dfrac{\sqrt{x}}{2}$

3. The graph drawn in the thick line is a transformation of the graph drawn in the thin line. Write an equation for each graph drawn in the thick line.

a)

$y = 3 - (x + 2)^2$

b)

$y = (x + 3)(x - 2)(x - 4)$

c)

$y = \dfrac{x + 2}{x + 3}$

d)

$y = \sqrt{-x}$
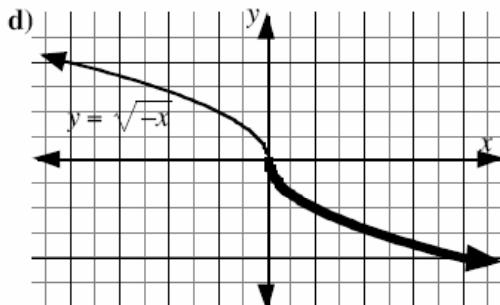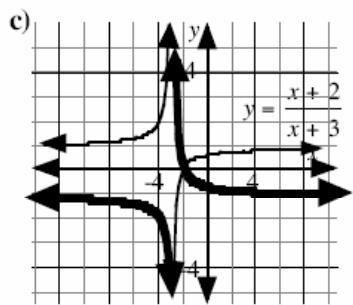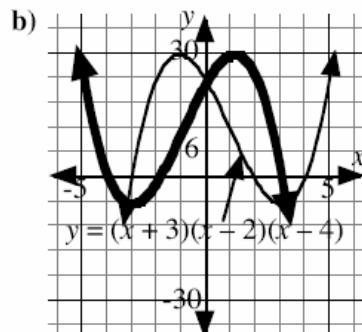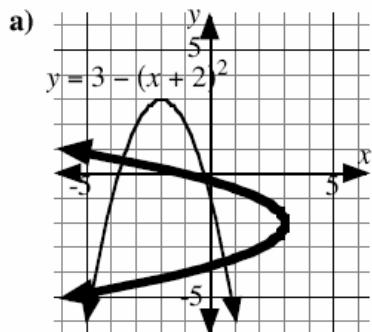
4.a) Sketch the graph of $f(x) = (x - 1)^2$.

 b) Write the equation for:

   i) $y = -f(x)$ **ii**) $y = f(-x)$

   iii) $x = f(y)$

 c) Sketch each graph in b) and state whether the graph represents a function.

   i) $y = -f(x)$ **ii**) $y = f(-x)$ **iii**) $x = f(y)$

5.a) Sketch the graph the semi-circle of $f(x) = \sqrt{16 - x^2}$.

b) Write the equation for:
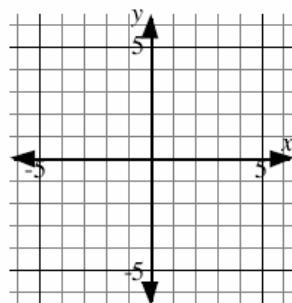
   i) $y = -f(x)$ **ii)** $y = f(-x)$

   iii) $x = f(y)$

c) Sketch each graph in b) and state whether the graph represents a function.
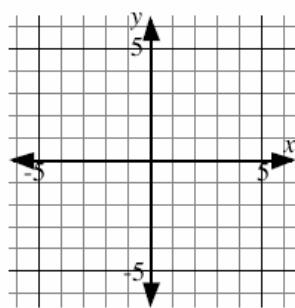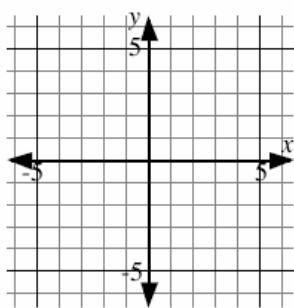
   i) $y = -f(x)$ **ii)** $y = f(-x)$ **iii)** $x = f(y)$
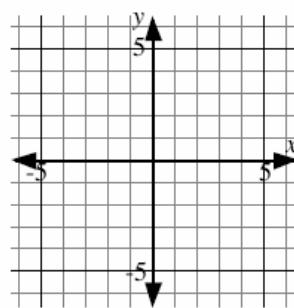
d) State the domain and range of each graph in c)

	i)	**ii)**	**iii)**
Domain			
Range			

Multiple Choice **6.** The graph of $y = 2x^5$ is transformed to the graph of $y = -2x^5$.
Consider the following statements about the transformed graph.

   i. It is a reflection of the original graph in the x-axis.
   ii. It is a reflection of the original graph in the y-axis.
   iii. It is a reflection of the original graph in the line $y = x$.

How many of the above statements are false?

   A. 0   **B.** 1   **C.** 2   **D.** 3

7. How could the graph of $y = 2x^3 + 1$ be used to graph $y = -2x^3 + 1$?
   A. Vertical translation of $y = 2x^3 + 1$.
   B. Reflection of $y = 2x^3 + 1$ in the line $y = x$.
   C. Reflection of $y = 2x^3 + 1$ in the x-axis.
   D. Reflection of $y = 2x^3 + 1$ in the y-axis.

Numerical Response **8.** The graphs below represent transformations of the graph of $f(x) = x^2$.

1.
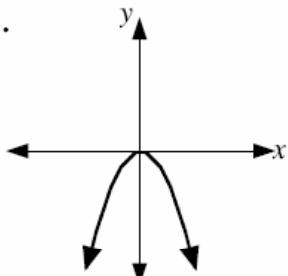
2.
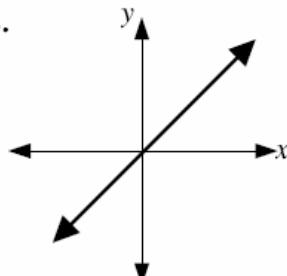
3.
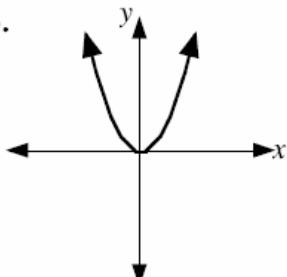
4.
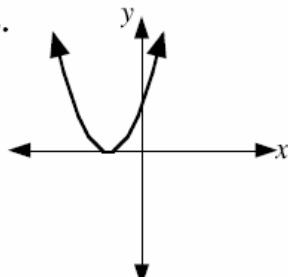
Match each of the graphs to the statement below.

The graph represents $f(-x)$ _____

The graph represents $f(x + 2)$ _____

The graph represents $f^{-1}(f(x))$ _____

The graph represents $-f(x)$ _____

Answer Key

1.a) $x = \dfrac{1}{y}$ or $y = \dfrac{1}{x}$ **b)** $y = -x^3 - x$ **c)** $y = -|x|$

   d) $x = \sqrt{y - 2}$ or $y = x^2 + 2, x \geq 0$ **e)** $y = x^2 + 1$ **f)** $y = -\cos x$

2.a) reflection in the x-axis   **b)** reflection in the y-axis   **c)** reflection in the line $y = x$

   d) reflection in the y-axis   **e)** reflection in the x-axis   **f)** reflection in the line $y = x$

3.a) $x = 3 - (y + 2)^2$ or $y = \pm \sqrt{3 - x} - 2$

   b) $y = (-x + 3)(-x - 2)(-x - 4)$ or $y = -(x - 3)(x + 2)(x + 4)$ **c)** $y = -\dfrac{x + 2}{x + 3}$ **d)** $y = -\sqrt{x}$

4.a) parabola opening up with vertex $(1, 0)$

   b) **i)** $y = -(x - 1)^2$ **ii)** $y = (-x - 1)^2$ or $y = (x + 1)^2$ **iii)** $x = (y - 1)^2$

   c) **i)** parabola opening down with vertex $(1, 0)$. Is a function.   **ii)** parabola opening up with
         vertex $(-1, 0)$. Is a function.   **iii)** parabola opening right with vertex $(0, 1)$. Is not a function.

5.a) top half of a circle with centre at the origin and radius 4

   b) **i)** $y = -\sqrt{16 - x^2}$ **ii)** $y = \sqrt{16 - x^2}$ **iii)** $x = \sqrt{16 - y^2}$

   c) **i)** bottom half of a circle with centre at the origin and radius 4. Is a function.
       ii) top half of a circle with centre at the origin and radius 4. Is a function.
       iii) right half of a circle with centre at the origin and radius 4. Is not a function.

   d) **i)** Domain: $\{x \mid -4 \leq x \leq 4, x \in \Re\}$ Range: $\{y \mid -4 \leq y \leq 0, y \in \Re\}$

       ii) Domain: $\{x \mid -4 \leq x \leq 4, x \in \Re\}$ Range: $\{y \mid 0 \leq y \leq 4, y \in \Re\}$

       iii) Domain: $\{x \mid 0 \leq x \leq 4, x \in \Re\}$ Range: $\{y \mid -4 \leq y \leq 4, y \in \Re\}$

6. B   **7.** D   **8.** 3421