Transformations Lesson #8:
Expansions and Compressions about the x- or y-axis Part 2

> **Warm-Up**

In the previous lesson we had the following note:

$y = af(x)$ can be written as $\dfrac{1}{a}y = f(x)$.

Given the function $y = f(x)$:

- replacing x with bx, $(i.e.\ x \to bx)$ describes a horizontal stretch about the y-axis.
    i.e. $y = f(bx)$ describes a horizontal stretch.

- replacing y with $\dfrac{1}{a}y$, $(i.e.\ y \to \dfrac{1}{a}y)$ describes a vertical stretch about the x-axis.
    i.e. $\dfrac{1}{a}y = f(x)$ or $y = af(x)$ describes a vertical stretch.

In general, if $\dfrac{1}{a}y = f(bx)$ or $y = af(bx)$, then for:

> $a > 1$ there is a vertical expansion
> $0 < a < 1$ there is a vertical compression
> $a < 0$ there is also a reflection in the x-axis
> $b > 1$ there is a horizontal compression
> $0 < b < 1$ there is a horizontal expansion
> $b < 0$ there is also a reflection in the y-axis

Class Ex. #1   Write the equation of the image of:

a) $y = x^2$ after a horizontal compression about the y-axis by a factor of $\dfrac{3}{4}$. $b = \dfrac{4}{3}$

$$y = \left(\tfrac{4}{3}x\right)^2 \Rightarrow y = \tfrac{16}{9}x^2$$

b) $y = \sqrt{x} - 3$ after a horizontal expansion by a factor of 4 about the y-axis and a vertical expansion by a factor of 2 about the x-axis. $a = 4 \quad b = \tfrac{1}{2}$

$$y = 4\left(\sqrt{\tfrac{1}{2}x} - 3\right) \Rightarrow y = 4\sqrt{\tfrac{1}{2}x} - 12$$

c) $y = 3x + 7$ after a vertical compression about the x-axis by a factor of $\dfrac{1}{3}$ and a reflection in the x-axis. $a = \tfrac{1}{3} \quad -f(x)$

$$y = -\tfrac{1}{3}(3x + 7) \Rightarrow y = -1x - \tfrac{7}{3}$$

Class Ex. #2   Describe how the graph of the second function compares to the graph of the first function.

a) $y = f(x)$

$y = f\left(\frac{1}{2}x\right)$ $b = \frac{1}{2}$

hor exp by factor of 2

b) $y = 2^x$

$y = 2^{3x}$ $b = 3$

hor comp by factor of $\frac{1}{3}$

c) $y = |x|$

$y = -2|x|$ $a = -2$

vert exp by factor of 2
reflection in the x axis

d) $y = |x|$

$y = |-2x|$ $b = -2$

hor comp by factor of $\frac{1}{2}$
reflection in the y-axis

e) $y = |x|$

$y = 2\left|\frac{1}{3}x\right|$ $a = 2$ $b = \frac{1}{3}$
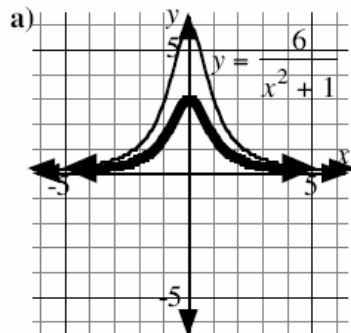
hor exp by factor of 3
vert exp by factor of 2

f) $y = x^3$

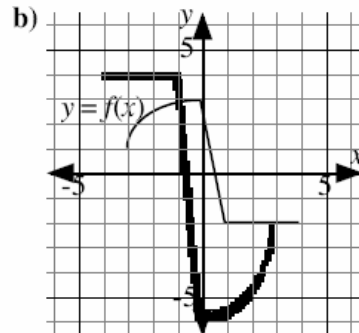$\frac{3y}{3} = \frac{x^3}{3}$ $y = \frac{1}{3}x^3$ $a = \frac{1}{3}$

vert comp by a factor of $\frac{1}{3}$
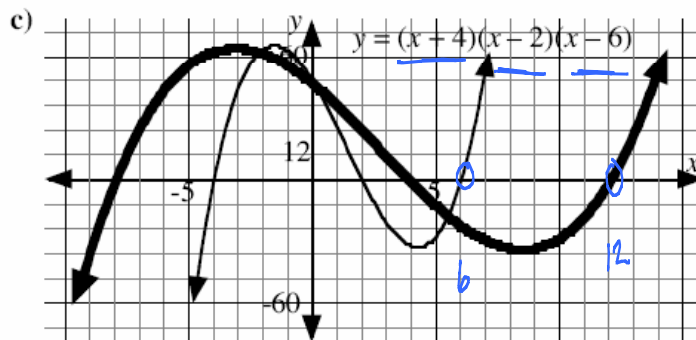

Class Ex. #3   The function represented by the thick line is a stretch of the function represented by the thin line. Write an equation for each function represented by the thick line.

a)

$y = \frac{}{x^2 + 1}$

Vert factor $\frac{1}{2}$

$a = \frac{1}{2}$

$y = \frac{1}{2}\left(\frac{1}{x^2+1}\right)$

$y = \frac{3}{x^2+1}$

b)

$y = f(x)$

Vert factor -2
reflection in the x axis

$y = -2f(x)$

c)

$y = (x+4)(x-2)(x-6)$

hor factor 2

$b = \frac{1}{2}$

$y = \left(\frac{1}{2}x-4\right)\left(\frac{1}{2}x-2\right)\left(\frac{1}{2}x-6\right)$

$\frac{1}{2}(x-8)\frac{1}{2}(x-4)\frac{1}{2}(x-12)$

$\frac{1}{2}(x-8)(x-4)(x-12)$

Class Ex. #4

A polynomial function has the equation $P(x) = (x - 4)(x + 3)(x + 6)$. Determine the zeros and the y-intercept if the following transformations are applied.

a) $y = -3P(x)$

$$y = -3\left[(x-4)(x+3)(x+6)\right]$$

zero's $\quad x-4=0 \quad x+3=0 \quad x+6=0$

$$x=4 \qquad x=-3 \qquad x=-6$$

yint $\quad -3\left[(0-4)(0+3)(0+6)\right]$

$$-3(-72) = 216$$

b) $y = P\left(-\dfrac{1}{2}x\right)$

$$y = \left(-\tfrac{1}{2}x-4\right)\left(-\tfrac{1}{2}x+3\right)\left(-\tfrac{1}{2}x+6\right)$$

zero's $\quad -\tfrac{1}{2}x-4=0 \quad -\tfrac{1}{2}x+3=0 \quad -\tfrac{1}{2}x+6=0$

$(-2)-\tfrac{1}{2}x=4(-2) \quad -\tfrac{1}{2}x=-3 \quad -\tfrac{1}{2}x=-6$

$$x=-8 \qquad x=6 \qquad x=12$$

yint $= \left(-\tfrac{1}{2}(0)-4\right)\left(-\tfrac{1}{2}(0)+3\right)\left(-\tfrac{1}{2}(0)+6\right)$

$$= (-4)(3)(6) = -72$$

Complete Assignment Questions #1 - #6

Assignment

1. Write the equation of the image of:

   a) $y = |x + 1|$ after a vertical compression about the x-axis by a factor of $\dfrac{7}{9}$.

   b) $y = 2^x$ after a horizontal expansion by a factor of 3 about the y-axis.

   c) $y = \sqrt{x - 2}$ after a vertical expansion about the x-axis by a factor of 4 and a reflection in the x-axis.

   d) $y = \sqrt{x - 2}$ after a horizontal expansion about the y-axis by a factor of 4 and a reflection in the y-axis.

   e) $y = \sin x°$ after a horizontal compression about the y-axis by a factor of $\dfrac{3}{4}$ and a vertical compression about the x-axis by a factor of $\dfrac{1}{2}$.

   f) $y = 2x - 11$ after a horizontal compression about the y-axis by a factor of $\dfrac{1}{3}$ and a reflection in the y-axis.

g) $y = \dfrac{1}{x + 3}$ after a horizontal expansion about the y-axis by a factor of 3, a vertical expansion about the x-axis by a factor of 2 and a reflection in the y-axis.

h) $y = \dfrac{1}{x} + 3$ after a vertical compression about the x-axis by a factor of $\dfrac{1}{2}$, a horizontal compression about the y-axis by a factor of $\dfrac{1}{4}$, and a reflection in both the x-axis and y-axis.

2. Describe how the graph of the second function compares to the graph of the first function.

a) $y = \sqrt{x}$

$y = \sqrt{\dfrac{1}{3}x}$

b) $y = x^4$

$\dfrac{1}{4}y = x^4$

c) $y = 5x + 10$

$y = 5\left(-\dfrac{1}{4}x\right) + 10$

d) $y = \cos x°$

$y = 3\cos 2x°$
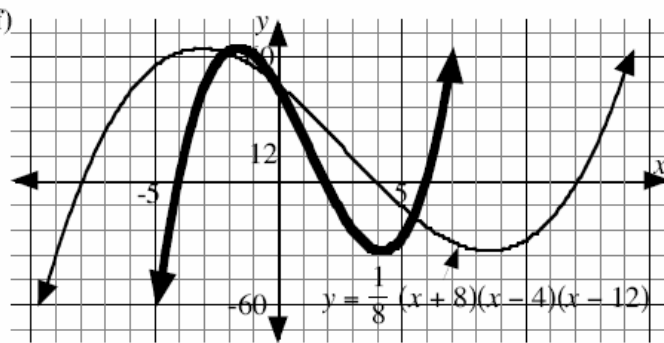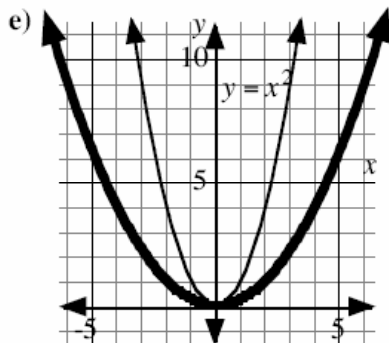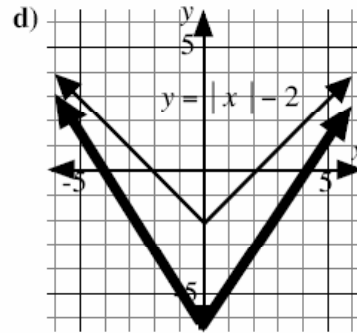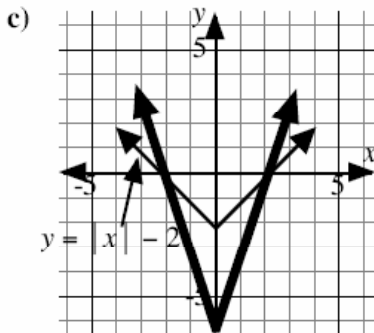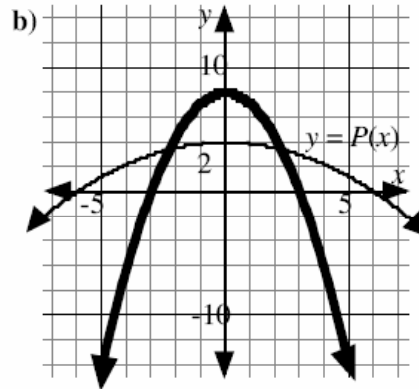
e) $y = 5^x$

$y = 5^{0.5x}$

f) $y = 6x - x^2$

$y = 6(2x) - (2x)^2$

g) $y = \dfrac{1}{x + 1}$

$y = \dfrac{2}{x + 1}$

h) $y = \dfrac{1}{x + 1}$

$3y = \dfrac{1}{x + 1}$

i) $y = \dfrac{1}{x + 1}$

$y = -\dfrac{4}{2x + 1}$

3. The function represented by the thick line is a stretch of the function represented by the thin line. Write an equation for each function represented by the thick line.

a)

b)

$y = P(x)$

$y = P(x)$

c)

$y = \left| x \right| - 2$

d)

$y = \left| x \right| - 2$

e)

$y = x^2$

f)

$y = \frac{1}{8}(x + 8)(x - 4)(x - 12)$

g)

h)
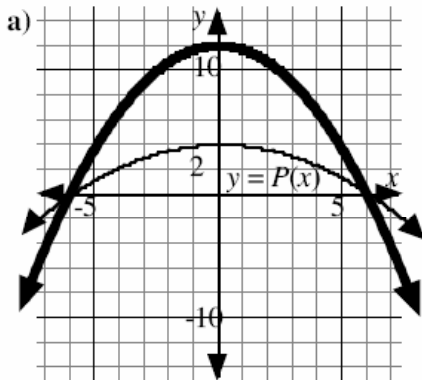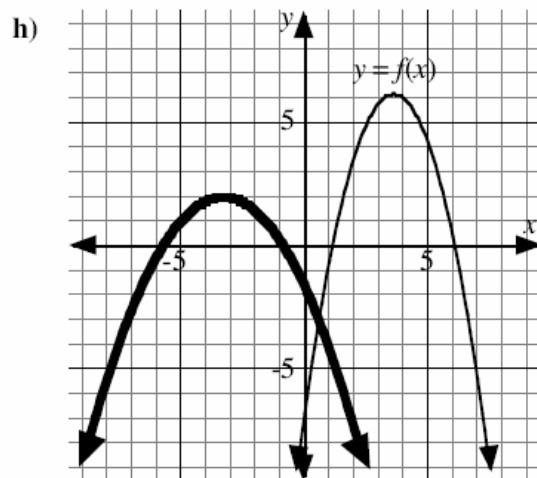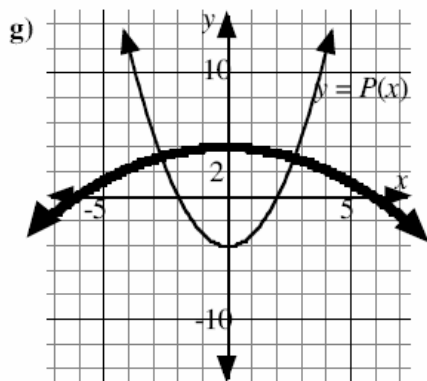
4. A polynomial function has the equation $P(x) = (x - 5)(x - 2)(x + 1)$. Determine the zeros and the y-intercept if the following transformations are applied.

   a) $y = 4P(x)$

   b) $y = P(4x)$

   c) $y = -\dfrac{1}{4}P(x)$

   d) $y = P\left(-\dfrac{1}{4}x\right)$

5. A polynomial function has the equation $P(x) = x(x - 2)^2$. Determine the zeros and the y-intercept if the following transformations are applied.

   a) $y = 7P(x)$

   b) $y = P\left(\dfrac{4}{3}x\right)$

Multiple Choice **6.** How is the graph of $\frac{1}{4}y = x^2$ related to the graph of $y = x^2$?

   A. $y = x^2$ has been expanded horizontally about the y-axis by a factor of 4.

   B. $y = x^2$ has been compressed horizontally about the y-axis by a factor of $\frac{1}{4}$.

   C. $y = x^2$ has been expanded vertically about the x-axis by a factor of 4.

   D. $y = x^2$ has been compressed vertically about the x-axis by a factor of $\frac{1}{4}$.

Answer Key

1. **a)** $y = \frac{7}{9}|x + 1|$ **b)** $y = 2^{\frac{1}{3}x}$ **c)** $y = -4\sqrt{x - 2}$ **d)** $y = \sqrt{-\frac{1}{4}x - 2}$

   e) $y = \frac{1}{2}\sin\frac{4}{3}x°$ **f)** $y = -6x - 11$ **g)** $y = \dfrac{2}{-\frac{1}{3}x + 3}$ or $y = -\dfrac{6}{x - 9}$ **h)** $y = \frac{1}{8x} - \frac{3}{2}$

2. **a)** horizontal expansion about the y-axis by a factor of 3
   b) vertical expansion about the x-axis by a factor of 4
   c) horizontal expansion about the y-axis by a factor of 4 and a reflection in the y-axis
   d) vertical expansion about the x-axis by a factor of 3 and a horizontal compression

      about the y-axis by a factor of $\frac{1}{2}$

   e) horizontal expansion about the y-axis by a factor of 2

   f) horizontal compression about the y-axis by a factor of $\frac{1}{2}$

   g) vertical expansion about the x-axis by a factor of 2

   h) vertical compression about the x-axis by a factor of $\frac{1}{3}$

   i) vertical expansion about the x-axis by a factor of 4, horizontal compression

      about the y-axis by a factor of $\frac{1}{2}$, and a reflection in the x-axis

3. **a)** $y = 3P(x)$ **b)** $y = 2P(2x)$ **c)** $y = 3|x| - 6$ **d)** $y = 3\left|\frac{1}{2}x\right| - 6$

   e) $y = \frac{1}{4}x^2$ **f)** $y = (x + 4)(x - 2)(x - 6)$ **g)** $y = -P\left(\frac{1}{3}x\right)$ **h)** $y = \frac{1}{3}f(-x)$

4. **a)** zeros: $-1, 2, 5$ **b)** zeros: $-\frac{1}{4}, \frac{1}{2}, \frac{5}{4}$ **c)** zeros: $-1, 2, 5$ **d)** zeros: $4, -8, -20$

      y-intercept: 40 y-intercept: 10 y-intercept: $-\frac{5}{2}$ y-intercept: 10

5. **a)** zeros: $0, 2$ y-intercept $= 0$ **b)** zeros: $0, \frac{3}{2}$ y-intercept $= 0$

6.   C