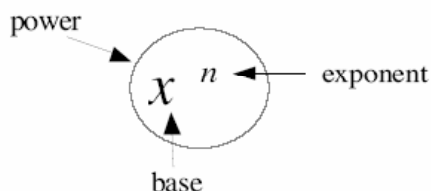# Exponential and Logarithmic Functions Lesson #1:
Review of Exponents

Warm-Up | *Review of Exponent Laws*

The parts of the power are listed below

power

x^n ← exponent

base

Complete the following:

Product Law $\quad x^m x^n = x^{m+n}$

Quotient Law $\quad x^m \div x^n = x^{m-n}$

Power of a Power $\quad (x^m)^n = x^{mn}$

Power of a Product $\quad (xy)^m = x^m y^m$

Power of a Quotient $\quad \left(\dfrac{x}{y}\right)^m = \dfrac{x^m}{y^m} \quad , y \neq 0$

Integral Exponent Rule $\quad x^{-m} = \dfrac{1}{x^n} \quad$, where $x \neq 0$

Rational Exponents $\quad x^{\frac{m}{n}} = \sqrt[n]{x^m} \text{ or } \left(\sqrt[n]{x}\right)^m$

Class Ex. #1 — Write each expression without brackets and with positive exponents.

a) $\dfrac{1}{2}y^{-6}$

$= \dfrac{1}{2y^6}$

b) $\dfrac{5x^{-3}}{y^{-2}}$

$= \dfrac{5y^2}{x^3}$

c) $(4x^3y)(2x^{-4}y^2)$

$= 8x^{-1}y^3 = \dfrac{8y^3}{x}$

d) $\dfrac{24m^5p^{-3}q^4}{-4m^4p^2q^{-2}}$

$= -6m^1p^{-5}q^6$

$= \dfrac{-6mq^6}{p^5}$

e) $(3x^2y^3)^3$

$= 27x^6y^9$

f) $\dfrac{12b^{-\frac{1}{2}}}{3b}$

$-\dfrac{1}{2} - \dfrac{2}{2} = \dfrac{-3}{2}$

$= 4b^{-\frac{3}{2}}$

$= \dfrac{4}{b^{\frac{3}{2}}}$

Class Ex. #2   Find the exact value of the following. Verify with a calculator.

a) 9^{-3} **b)** $81^{\frac{3}{4}}$ **c)** 25^0 **d)** $16^{-\frac{1}{2}}$

a) $= \dfrac{1}{9^3} = \dfrac{1}{729}$

b) $= \sqrt[4]{81}^{\,3}$
$= 3^3$
$= 27$

c) $= 1$

d) $= \dfrac{1}{\sqrt{16}}$
$= \dfrac{1}{4}$

Class Ex. #3   Simplify the expression $\dfrac{p^{-1}q^{\frac{1}{2}}}{4p^2r^2}$ and evaluate when $p = 2$, $q = 3$, and $r = -5$.

$\dfrac{p^{-3}q^2}{4r^2} = \dfrac{q^2}{4p^3r^2} = \dfrac{(3)^2}{4\cdot(2)^3\cdot(-5)^2} = \dfrac{9}{4\cdot 8\cdot 25} = \dfrac{9}{800}$

Class Ex. #4   Simplify the following. Write the answers with positive exponents.

a) $(4xy^{-2})^{-3}$

$= 4^{-3}x^{-3}y^6$

$= \dfrac{y^6}{64x^3}$

b) $\dfrac{-a^5(b^{-3})^2}{b^{-1}(a^4)^2}$

$= \dfrac{-a^{5}\cdot b^{-6-(-1)}}{a^8\cdot b^{-1}}$

$= -a^{-3}b^{-5} = \dfrac{-1}{a^3b^5}$

c) $\left(\dfrac{3x^3}{4y^{-2}}\right)^{-2}$

$= \left(\dfrac{4y^{-2}}{3x^3}\right)^2$

$= \dfrac{16y^{-4}}{9x^6} = \dfrac{16}{9x^6y^4}$

d) $\left(\dfrac{-8x^8y^5}{25x^2y}\right)^{-1}\left(\dfrac{5x^3y^9}{-4xy^5}\right)^2$

$\left(\dfrac{-8x^6y^4}{25}\right)\left(\dfrac{5x^2y^4}{-4}\right)^2$

$\left(\dfrac{-8x^6y^4}{25}\right)\left(\dfrac{25x^4y^8}{16}\right)$

$-\dfrac{1}{2}x^{10}y^{12}$

BEDMAS
Brackets first
Exponents
$\times/\div$
$+/-$

Changing Base

Class Ex. #5   Convert each of the following to the base indicated.

a) 9^{2x} to base 3

$$= \left(3^2\right)^{2x}$$

$$= 3^{4x}$$

b) 125^{2-x} to base 5

$$= \left(5^3\right)^{(2-x)}$$

$$= 5^{6-3x}$$

c) $8 \cdot 16^x$ to base 2

$$= 2^3 \cdot \left(2^4\right)^x$$

$$= 2^3 \cdot 2^{4x} = 2^{3+4x}$$

$$x^{-n} = \frac{1}{x^n}$$

d) $\dfrac{1}{512^{3x}}$ to base 2

$$= \left(2^{-9}\right)^{3x}$$

$$= 2^{-27x}$$

e) $\left(\dfrac{16}{81}\right)^{x+5}$ to base $\dfrac{2}{3}$

$$= \left(\frac{2}{3}\right)^{4(x+5)}$$

$$= \left(\frac{2}{3}\right)^{4x+20}$$

Complete Assignment Questions #1 - #10

Assignment

1. Write each expression without brackets and with positive exponents.

a) $4xy^{-3}$

b) $\dfrac{15y^{-3}}{5y}$

c) $(3x^3y)(5x^{-2}y^4)$

d) $\dfrac{24p^{-8}}{16p^{-3}}$

e) $\dfrac{2}{a^{-\frac{1}{3}}}$

f) $(2x^{-2})^3$

2. Find the exact value of the following:

a) 5^{-2} b) $27^{\frac{4}{3}}$ c) $\left(\dfrac{4}{9}\right)^{-\frac{3}{2}}$ d) $125^{\frac{1}{3}} - 10^0(64)^{\frac{2}{3}}$ e) $\left(\dfrac{1}{4}\right)^{-2}$

3. Evaluate the following expressions for $a = 1$, $b = -2$, and $c = 3$.

a) $(a^{-2}b^{-4})(a^2b^{-5})$ b) $\dfrac{a^{-1}b^3c^{-2}}{abc}$ c) $\dfrac{a^{-1}+b^{-1}}{c^{-1}+c^{-2}}$

4. Simplify the following. Write the answers with positive exponents.

a) $\dfrac{x^5 y^{-1}}{x^2 y^{-4}}$ b) $\dfrac{-a^{-2}(b^{-1})^2}{b^3(-a^4)^2}$

c) $\left(\dfrac{5x^3}{2y^4}\right)^{-3}$ d) $(4m^2n)^{-1} \times 2mn^5$

e) $\left(\dfrac{-8x^8y^5}{24x^2y}\right)\left(\dfrac{15x^3y^9}{18xy^5}\right)^2$ f) $\dfrac{3x^2y^0z^{-4}}{(2xyz)^3}$

5. Convert each of the following to the base indicated.

    a) 32^x to base 2
    b) 81^{x-2} to base 3
    c) $\dfrac{1}{64^{2x}}$ to base 4

    d) $\left(\dfrac{1}{16}\right)^{x+1}$ to base 2
    e) $\left(\dfrac{25}{49}\right)^{3x}$ to base $\dfrac{5}{7}$
    f) $\left(\dfrac{27}{64}\right)^{x+2}$ to base $\dfrac{4}{3}$

6. Convert each of the following to the base indicated.

    a) $2 \cdot 4^x$ to base 2
    b) $9 \cdot 27^{x-1}$ to base 3
    c) $\dfrac{1}{4} \cdot \left(\dfrac{1}{16}\right)^{4-x}$ to base 4

Multiple Choice

7. $(4x^{-3}y^5)^2$ is equal to

    A. $\dfrac{16y^{10}}{x^6}$

    B. $\dfrac{4y^{10}}{x^6}$

    C. $\dfrac{16y^{10}}{x^3}$

    D. $\dfrac{16x^6}{y^{10}}$

8. $\dfrac{(2a^2b)^{-3}}{(ab^2)^{-4}}$ is equal to

    A. $\dfrac{b^5}{2a^2}$

    B. $\dfrac{b^5}{8a^2}$

    C. $\dfrac{8a^2}{b^5}$

    D. $\left(\dfrac{2a}{b}\right)^{-7}$

9. $(36x^{-4})^{-\frac{1}{2}}$ is equal to

     A. $\dfrac{6}{x^2}$

     B. $-18x^2$

     C. $\dfrac{x^2}{6}$

     D. $\dfrac{x^{-4.5}}{6}$

10. $\dfrac{(64p^2q^{-\frac{2}{3}})^{-\frac{1}{2}}}{(p^5q^{10})^{-\frac{1}{5}}}$ is equal to

     A. $\dfrac{8}{p^3}$

     B. $\dfrac{-32}{q^{\frac{4}{3}}}$

     C. $\dfrac{q^{\frac{7}{3}}}{8p}$

     D. $\dfrac{q^{\frac{7}{3}}}{8}$

Answer Key

1.   a) $\dfrac{4x}{y^3}$ b) $\dfrac{3}{y^4}$ c) $15xy^5$ d) $\dfrac{3}{2p^5}$ e) $2a^{\frac{1}{3}}$ f) $\dfrac{8}{x^6}$

2.   a) $\dfrac{1}{25}$ b) 81 c) $\dfrac{27}{8}$ d) -11 e) 16

3.   a) $-\dfrac{1}{512}$ b) $\dfrac{4}{27}$ c) $\dfrac{9}{8}$

4.   a) x^3y^3 b) $\dfrac{-1}{a^{10}b^5}$ c) $\dfrac{8y^{12}}{125x^9}$ d) $\dfrac{n^4}{2m}$ e) $\dfrac{-25x^{10}y^{12}}{108}$ f) $\dfrac{3}{8xy^3z^7}$

5.   a) 2^{5x} b) 3^{4x-8} c) 4^{-6x} d) 2^{-4x-4} e) $\left(\dfrac{5}{7}\right)^{6x}$ f) $\left(\dfrac{4}{3}\right)^{-3x-6}$

6.   a) 2^{2x+1} b) 3^{3x-1} c) 4^{2x-9}

7.   A   8.   B   9.   C   10.   D