

Exponential and Logarithmic Functions Lesson #2: Solving Equations Involving Exponents

Warm-Up #1

Review

Simplify $(9^{2x+3} \div 27^{3x-1}) \times 81^{x-1}$ by converting each term to a common base.

$$\begin{aligned}
 &= \left(3^{2(2x+3)} \div 3^{3(3x-1)} \right) \times 3^{4(x-1)} \\
 &= 3^{4x-9x+6-(-3)} \times 3^{4x-4} = 3^{-5x+9} \times 3^{4x-4} \\
 &= 3^{4x+6} \div 3^{9x-3} \times 3^{4x-4} = 3^{-5x+9} \times 3^{4x-4} = 3^{-5x+4x+9+(-4)} \\
 &= 3^{-1x+5} \text{ or } 3^{5-x}
 \end{aligned}$$

Solving Equations with Rational Exponents

Use the following procedure to solve an equation where the exponent is rational:

- Raise both sides to the reciprocal power of the exponent
- Simplify and solve for the variable.



Solve for x in the following.

a) $x^{-\frac{4}{3}} = 81$ $x^{\frac{4}{3} \cdot \frac{3}{4}} = 81^{1 \cdot \frac{3}{4}}$ b) $(3x-5)^{\frac{3}{2} \cdot \frac{2}{3}} = 27^{\frac{2}{3}}$ $3x-5+5 = 9+5$

$$\begin{aligned}
 x^{\frac{4}{3} \cdot \frac{3}{4}} &= 81^{\frac{3}{4}} & 3x-5 &= \sqrt[3]{27}^2 & \frac{3x}{3} &= \frac{14}{3} \\
 x &= 81^{\frac{3}{4}} = \frac{1}{\sqrt[4]{81}^3} = \frac{1}{3^3} = \frac{1}{27} & 3x-5 &= 3^2 & x &= \frac{14}{3} \text{ or } 4\frac{2}{3} \\
 & & 3x-5 &= 9 & &
 \end{aligned}$$

Solving Exponential Equations with a Common Base

An **exponential equation** is an equation where the variable is in the exponent.

Use the following procedure to solve an equation where the variable is in the exponent:

- Write both sides of the equation in the same base
- Equate the exponents on both sides of the equation
- Determine the value of the variable.



Solve for x in

a) ~~$5^{2x+3} = 5^7$~~ b) $7^{x-2} = 343$ c) $3^{5x-1} = 81^{3x}$ d) $3^x = 27\sqrt{3}$

$$\begin{aligned}
 2x+3 &= 7 & \cancel{7}^{x-2} &= \cancel{7}^3 & \cancel{3}^{5x-1} &= \cancel{3}^{4(3x)} & 3^x &= 3^3 \cdot 3^{\frac{1}{2}} \\
 2x+3-3 &= 7-3 & x-2 &= 3 & 5x-1 &= 12x & \cancel{3}^x &= \cancel{3}^{\frac{7}{2}} \\
 2x &= 4 & x &= 5 & -5x-1 &= -5x & x &= \frac{7}{2} \text{ or } 3\frac{1}{2} \\
 \frac{2x}{2} &= \frac{4}{2} & & & -1 &= \frac{7x}{7} & & \\
 x &= 2 & & & x &= -\frac{1}{7} & &
 \end{aligned}$$

d) $27^{x-2} = \frac{1}{81^{x+3}}$

~~3~~ $3(x-2) = \frac{1}{\del{3}^{-4(x+3)}}$

$3x-6 = -4x-12$
 $+4x \quad +4x$

$7x-6 = -12$
 $+6 \quad +6$

$\frac{-7x}{-7} = \frac{-8}{-7} \quad x = \frac{8}{7} \text{ or } 1\frac{1}{7}$

e) $\left(\frac{125}{216}\right)^{\frac{-x}{4}} = \left(\frac{6}{5}\right)^{3x-3}$

~~$\left(\frac{6}{5}\right)^{-3\left(\frac{-x}{4}\right)} = \left(\frac{6}{5}\right)^{3x-3}$~~

$\frac{3}{4}x-3x = 3x-3$
 $-3x \quad -3x$

$\frac{3}{4}x - \frac{12x}{4} = -3$

$\left(\frac{-4}{9}\right) - \frac{9}{4}x = -3\left(\frac{-4}{9}\right)$

$x = \frac{12}{9} = \frac{4}{3} = 1\frac{1}{3}$

Complete Assignment Questions #1 - #8

Assignment

1. Simplify.

a) $49^{x-1} \times 7^{2x-3}$

b) $216^x + (1296^{5x-4} \times 36^{x+5})$

c) $64^{3x} \times 128^{x-1} + (32^{2x+3} + 8^{4x-1})$

2. Solve for x .

a) $x^{\frac{1}{2}} = 5$

b) $x^{-\frac{1}{2}} = 5$

c) $x^{\frac{1}{3}} = -5$

d) $4x^{-\frac{2}{3}} = 16$

3. Solve for x .

a) $2^x = 16\sqrt{2}$

b) $2^{-x} = 64$

c) $9^{3x+1} = 27^{3x}$

4. Solve for x .

a) $\left(\frac{4}{7}\right)^{5x} = \left(\frac{64}{343}\right)^{2x-1}$

b) $\left(\frac{125}{216}\right)^{-\frac{x}{2}} = \left(\frac{6}{5}\right)^{3x+2}$

c) $49\left(\frac{7}{12}\right)^{2x} = 144$

d) $\left(\frac{9}{4}\right)^{x+3} = \left(\frac{8}{27}\right)^{-5}$

e) $2^{x-1} = (128^x)(2^x)$

f) $2(6^{2x}) - 74(6^x) + 72 = 0$
(Hint: Write as a quadratic equation
with the variable as 6^x)

g) $\left(\frac{1}{4}\right)^{x-12} = (2)(32)^{2x+1}$

h) $\sqrt[3]{\frac{27^{2x-1}}{3^{x+1}}} = 9$

Multiple Choice

5. If $4^{2x-7} = \frac{1}{64}$, then the value of \sqrt{x} is

- A. 2
- B. $\sqrt{2}$
- C. $\sqrt{5}$
- D. $\frac{3}{2}$

Numerical Response

6. The solution to the equation $25^{x+1} = 5^{3(x-1)}$ to the nearest tenth, is $x = \underline{\hspace{2cm}}$.

7. The solution to the equation $\left(\frac{1}{8}\right)^{x-3} = (2)(16)^{2x+1}$, to the nearest hundredth, is $x = \underline{\hspace{2cm}}$.

8. The solution to the equation $8^{2x-1} = 16$, to the nearest tenth, is $x = \underline{\hspace{2cm}}$.

Answer Key

1. a) 7^{4x-5} b) 6^{-19x+6} c) 2^{27x-25}
 2. a) $x = 25$ b) $x = \frac{1}{25}$ c) $x = -125$ d) $x = \frac{1}{8}$
 3. a) $x = \frac{9}{2}$ b) $x = -6$ c) $x = \frac{2}{3}$
 4. a) $x = 3$ b) $x = -\frac{4}{3}$ c) $x = -1$ d) $x = \frac{9}{2}$ e) $x = -\frac{1}{7}$ f) $x = 0, x = 2$
 g) $x = \frac{3}{2}$ h) $x = 2$ 5. B 6. 5.0 7. 0.36 8. 1.2