Exponential and Logarithmic Functions Lesson #6:
Laws of Logarithms

Warm-Up #1

Evaluate the following.

i) $\log_2 16 + \log_2 8$

$4 + 3 = 7$

ii) $\log_2 [(16)(8)]$ $= \log_2 128$

$= 7$

What do you notice?

Warm-Up #2

Evaluate the following log equations

i) $\log_2 16 - \log_2 8$

$4 - 3 = 1$

ii) $\log_2 \frac{16}{8}$ $= \log_2 2$

$= 1$

What do you notice?

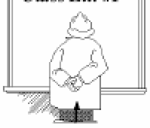
Product and Quotient Laws of Logarithms

Warm-Ups #1 and #2 are examples of the following laws

$\log_a (M \times N) = \log_a M + \log_a N$	**The Product Law**
$\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$	**The Quotient Law**

The laws of logarithms are identities which will be proved in Lesson 9.

These formulas are NOT on the formula sheet



Class Ex. #1

Evaluate the following using the product law or quotient law.

a) $\log_2 12 - \log_2 3$ $= \log_2 \frac{12}{3} = \log_2 4$

$= 2$

b) $\log_6 9 + \log_6 8 - \log_6 2$ $= \log_6 72 - \log_6 2 = \log_6 \frac{72}{2}$

$= \log_6 36 = 2$

c) $\log_5 10 + \log_5 75 - (\log_5 2 + \log_5 3)$

$= \log_5 750 - (\log_5 6)$

$= \log_5 \frac{750}{6} = \log_5 125 = 3$

d) $\log_2 2 + \log_2 3 - \log_2 6 - \log_2 8$

$- \log_2 2 + \log_2 3 - (\log_2 6 + \log_2 8)$

$= \log_2 6 - (\log_2 48)$

$= \log_2 \frac{6}{48} = \log_2 \frac{1}{8} = \log_2 2^{-3} = -3$

Class Ex. #2

The expression $\log_2 x + \log_2 2x - \log_2 x^2 - \log_2 y$ is equivalent to

A. $2 + \log y_2$
B. $1 + \log_2 y$
C. $2 - \log_2 y$
D. $1 - \log_2 y$

$= \log_2 2x^2 - (\log_2 x^2 + \log_2 y)$

$= \log_2 2x^2 - \log_2 x^2 y = \log_2 \frac{2x^2}{x^2 y} = \log_2 \frac{2}{y} = \log_2 2 - \log_2 y = 1 - \log_2 y$

Class Ex. #3

Evaluate.

a) $\log_2 15 + \log_2 14 - \log_2 105$

$= \log_2 210 - \log_2 105 = \log_2 \frac{210}{105} = \log_2 2 = 1$

b) $\log_4 2^8 + \log_4 \left(\frac{1}{8}\right)^2$

$\log_4 256 + \log_4 \frac{1}{64}$ or $= \log_4 (4)^4 + \log_4 (4)^{-3}$

$= \log_4 256 \cdot \frac{1}{64}$ $\quad = 4 + -3$

$= \log_4 4$ $\quad = 1$

$= 1$

Complete Assignment Questions #1 - #3

Warm-up #3

$* \; x^2 = x \cdot x$

Show that $2\log_3 x = \log_3 x^2$.

LS	RS
"	$\log_3 x + \log_3 x$
$2\log_3 x$	$2\log_3 x$
LS	= RS

The Power Law of Logarithms

Warm-Up #3 is an example of the power law of logarithms.

$\log_a M^n = n \log_a M$	**The Power Law**

This formula is NOT on the formula sheet

Class Ex. #4

Use the laws of logarithms to evaluate each of the following.

a) $\frac{1}{2}\log_2 16 - \frac{1}{3}\log_2 8$

$= \log_2 16^{\frac{1}{2}} - \log_2 8^{\frac{1}{3}}$

$= \log_2 4 - \log_2 2$

$= \log_2 \frac{4}{2} = \log_2 2 = 1$

b) $2 \log 5 + 2 \log 2$

$\log 5^2 + \log 2^2$

$\log 25 + \log 4$

$\log 100 = 2$

c) $3\log x - \log x^3$

$= \log x^3 - \log x^3$

$= \log \frac{x^3}{x^3}$

$= \log 1 = 0$

d) $\log_3 \frac{27^3}{81^4}$

$= \log_3 27^3 - \log_3 81^4$

$= \log_3 (3^3)^3 - \log_3 (3^4)^4$

$= \log_3 3^9 - \log_3 3^{16}$

$= 9 - 16 = -7$

Class Ex. #5

Simplify the following.

a) $\log_6 6^n$

$= n\log_b 6$

$= n\log_6 6^1$

$= n(1) = n$

b) $6^{\log_6 n}$

6^y

Replace: $\log_6 n = y$

$6^y = n$

$\therefore 6^{\log_6 n} = n$

Note

Class Ex. #5 is an example of the following logarithmic identities:

* • $\log_b b^n = n$ and • $b^{\log_b n} = n$ *

These identities follow from the fact that the logarithmic and exponential functions are inverses.

Class Ex. #6

Write the following expression as a single logarithm.

$$\log B + \log D - 5\log E - \log A^2 + \frac{1}{2}\log A$$

$= \log B + \log D + \frac{1}{2}\log A - 5\log E - \log A^2$

$= \log B + \log D + \log A^{\frac{1}{2}} - (\log E^5 + \log A^2)$

$= \log A^{\frac{1}{2}}BD - \log A^2E^5$

$= \log \frac{A^{\frac{1}{2}-2}BD}{A^2E^5} = \log \frac{A^{-\frac{3}{2}}BD}{E^5}$

$\frac{1}{2} - 2$

$\frac{1}{2} - \frac{4}{2} = -\frac{3}{2}$

Class Ex. #7

Write the following expression as a single logarithm.

$$\frac{3}{2}(\log_b x + 2\log_b y^4) - \frac{1}{2}\left(\log_b\sqrt{x} + \log_b y^{\frac{1}{3}}\right)$$

$= \frac{3}{2}\log_b x + \frac{3}{2}\log_b y^8 - \left(\frac{1}{2}\log_b x^{\frac{1}{2}} + \frac{1}{2}\log_b y^{\frac{1}{3}}\right)$

$\frac{3}{2}\times 8 = \frac{24}{2} = 12$

$= \log_b x^{\frac{3}{2}} + \log_b y^{12} - \left(\log_b x^{\frac{1}{4}} + \log_b y^{\frac{1}{6}}\right)$

$\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ $\frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}$

$= \log_b x^{\frac{3}{2}}y^{12} - \log_b x^{\frac{1}{4}}y^{\frac{1}{6}}$

$= \log_b \frac{x^{\frac{3}{2}-\frac{1}{4}}y^{12-\frac{1}{6}}}{x^{\frac{1}{4}}y^{\frac{1}{6}}}$

$\frac{3}{2} - \frac{1}{4} = \frac{6}{4} - \frac{1}{4} = \frac{5}{4}$

$12 - \frac{1}{6} = \frac{72}{6} - \frac{1}{6} = \frac{71}{6}$

$= \log_b x^{\frac{5}{4}}y^{\frac{71}{6}}$

Complete Assignment Questions #4 - #14

Assignment

1. Use the laws of logarithms to evaluate each of the following.

a) $\log_2 4 + \log_2 8$

b) $\log_4\left(\frac{7}{2}\right) - \log_4 56$

c) $\log_6 9 + \log_6 8 - \log_6 2$

d) $\log 2 + \log 10 - \log \frac{1}{5}$

2. Use the laws of logarithms to evaluate each of the following.

a) $\log_2\left(\frac{4}{3}\right) + \log_2 768$

b) $\log 8 + \log 5 - \log \frac{2}{5}$

c) $\log 3 + \log 4 + \log \frac{1}{2} + \log \frac{1}{6}$

d) $\log_3 3 + \log_3 2 - \log_3 27 - \log_3 6$

e) $\log_3 9 - \log_3\left(\frac{1}{3}\right)$

f) $\log_2(2^8) + \log_2\left(\frac{1}{8}\right)^2$

g) $\log_a 1 + \log_a 1 + \log_a 1$

3. Which of the following are true and which are false for logarithms to ***every base***?

a) $\log 2 + \log 3 = \log 5$ **b)** $\log 3 + \log 4 = \log 12$ **c)** $\log 8 = \log 4 + \log 2$

d) $\log 10 + \log 10 = \log 100$ **e)** $\log 2 \times \log 3 = \log 6$ **f)** $\dfrac{\log 8}{\log 2} = \log 4$

g) $\log 3^2 + \log 3^{-2} = 0$ **h)** $\log \dfrac{5}{3} = \dfrac{\log 5}{\log 3}$ **i)** $\log \dfrac{1}{8} = -\log 8$

4. Use the laws of logarithms to evaluate the following.

a) $\log_2 \sqrt[3]{16}$ **b)** $\log_3 27^{\frac{1}{2}}$

5. Write each expression as a single logarithm:

a) $\log x - 3 \log y - 2 \log z$ **b)** $\dfrac{1}{3} \log_a p + 3 \log_a q - 4 \log_a p$

6. Simplify the following:

a) $\log 2 + 2 \log 3 - \log 18$ **b)** $2\log_4 2 - 2\log_4 4 - \log_4 \dfrac{1}{4}$

7. a) Show that $\log 81 = 4 \log 3$.

b) Hence simplify: **(i)** $\log 81 - \log 27$ **(ii)** $\dfrac{\log 81}{\log 27}$

8. Which of the following are true and which are false for logarithms to ***every base***?

a) $\log 5^{-2} = -2 \log 5$

b) $\log 4 = \frac{2}{3} \log 8$

c) $\log 27 = \frac{3}{4} \log 81$

d) $\frac{1}{3} \log 11 = \log \frac{11}{3}$

e) $\log 5 = \frac{1}{2} \log 10$

f) $\frac{\log \sqrt{2}}{\log \sqrt{8}} = \frac{1}{3}$

g) $\log \frac{1}{5} - \log 5 = -\log 25$

h) $\log 2 - \log \sqrt{2} = \log\sqrt{2}$

9. Which is the greatest of $\frac{2}{3} \log 1, \frac{3}{4} \log 1, \frac{4}{3} \log 1$?

10. Simplify the following:

a) $\log x^4 - 3 \log x + \log \frac{1}{x}$

b) $\log x^{\frac{1}{2}} + \log y^{\frac{1}{2}} - \frac{1}{2} \log xy$

c) $\log_2\sqrt{6} - \frac{1}{2}\log_2 3$

d) $\frac{1}{2}\log_{10}10 + 3\log_{10}\sqrt{10}$

e) $\log_a a + \log_c a^5 - \log_c a$

f) $\frac{2}{3}\log_2 a - 5\log_2 b - \frac{1}{5}\log_2 c^3$

g) $\log_a y^{2x-3} + \log_a y^{5x-2} - \log_a y^{x-5} - 2\log_a y^{3x+1}$

11. Simplify.

a) $\log_5 5^7$ **b)** $10^{\log 6}$ **c)** $\ln e^4$ **d)** $\log_c c^t$

e) $e^{\ln 7}$ **f)** $(5^{\log_5 2})(5^{\log_5 3})$ **g)** $\dfrac{\left(\sqrt{2}^{\ \log_6 27}\right)\left(\sqrt{2}^{\ \log_6 16}\right)}{\sqrt{2}^{\ \log_6 12}}$

Multiple Choice **12.** The expression $3\log_x 4 + \log_x 8 - \frac{1}{4}\log_x 16$, where $x > 0$, is equal to

A. $\log_x 384$

B. $\frac{3}{4}\log_x 512$

C. $\log_x 256$

D. $\frac{1}{4}\log_x\left(\frac{1}{2}\right)$

13. If $X > 0$, $Y > 0$, and $Z = X^3Y$, then $\log Y$ is equal to

A. $3\log X - \log Z$

B. $\log Z - 3\log X$

C. $\log 3X - \log Z$

D. $\log Z - \log 3X$

Numerical Response

14. The value of $\dfrac{3^{\log_2 4\sqrt{5}}}{3^{\log_2 \sqrt{5}}}$ to the nearest tenth is _____ .

Answer Key

1. a) 5 b) –2 c) 2 d) 2

2. a) 10 b) 2 c) 0 d) –3 e) 3 f) 2 g) 0

3. a) F b) T c) T d) T e) F f) F g) T h) F i) T

4. a) $\frac{4}{3}$ b) $\frac{3}{2}$

5. a) $\log\left(\dfrac{x}{y^3z^2}\right)$ b) $\log_a \dfrac{q^3}{p^{\frac{11}{3}}}$ **6.** a) 0 b) 0

7. b) (i) log 3 (ii) $\frac{4}{3}$

8. a) T b) T c) T d) F e) F f) T g) T h) T

9. none because each of these equal zero.

10. a) 0 b) 0 c) $\frac{1}{2}$ d) 2 e) $1 + \log_c a^4$ f) $\log_2\left(\dfrac{a^{\frac{2}{3}}}{b^5c^{\frac{3}{5}}}\right)$

g) $-2\log_a y$ or $\log_a \dfrac{1}{y^2}$

11. a) 7 b) 6 c) 4 d) t e) 7 f) 6 g) 2

12. C **13.** B **14.** 9.0