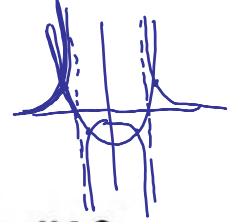


Trigonometry - Functions and Graphs Lesson #10:
Reciprocal and Absolute Value Trigonometric Functions

Warm-Up *Review of Reciprocal Functions*

Recall the properties of reciprocal functions by completing the following:

1. • When $f(x) = 0$, the graph of $y = \frac{1}{f(x)}$ may have a vertical asymptote.

• When $f(x)$ is positive, $\frac{1}{f(x)}$ is positive.

• When $f(x)$ is negative, $\frac{1}{f(x)}$ is negative.

2. • When $f(x) = 1$, $\frac{1}{f(x)} =$ 1. When $f(x) = -1$, $\frac{1}{f(x)} =$ -1.

• The invariant points for a reciprocal transformation can be found where the lines $y = \pm 1$ intersect the graphs of $f(x)$ and $\frac{1}{f(x)}$.

3. • When $f(x)$ increases over an interval, $\frac{1}{f(x)}$ decreases over the same interval.

• When $f(x)$ decreases over an interval, $\frac{1}{f(x)}$ increases over the same interval.

4. • When $f(x)$ approaches zero, $\frac{1}{f(x)}$ approaches $\pm\infty$ and the graph of $\frac{1}{f(x)}$ approaches a vertical asymptote.

• When $f(x)$ approaches $\pm\infty$, $\frac{1}{f(x)}$ approaches zero and the graph of $\frac{1}{f(x)}$ approaches a horizontal asymptote.

• Remember: $\sin^{-1} x$ does **NOT** mean $\frac{1}{\sin x}$. $\sin^{-1} x$ represents the <u>inverse</u> of the function $\sin x$. The reciprocal of $\sin x$ is $\csc x$.

• The above properties can be used as a general aid to sketch the reciprocal trigonometric functions.

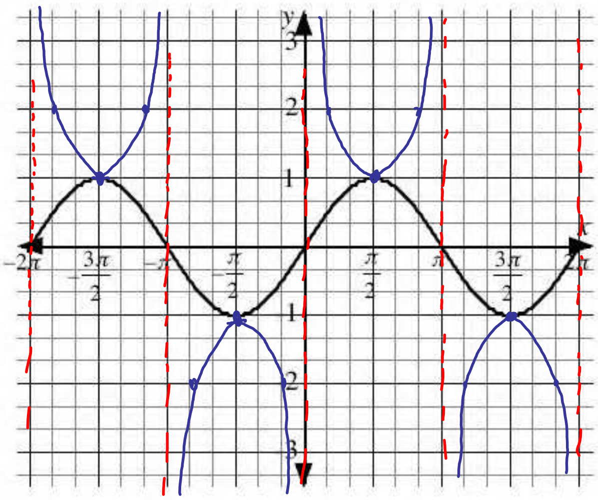
Sketching the Graph of a Reciprocal Trigonometric Function

Use the following general procedure to sketch the graph of a reciprocal trigonometric function.

1. Sketch the vertical asymptotes.
2. Mark the invariant points.
3. Where y approaches zero on the original graph, y approaches positive or negative infinity on the reciprocal graph.

Class Ex. #1

The graph of $y = \sin x$, $-2\pi \le x \le 2\pi$ is shown.



a) Graph $y = \csc x$, the reciprocal of $y = \sin x$, using the properties on the previous page.

b) State the equations of the asymptotes.

$x = -2\pi$, $x = -\pi$, $x = 0$

$x = \pi$, $x = 2\pi$

c) List the invariant points.

$\left(-\frac{3\pi}{2}, 1\right)$ $\left(-\frac{\pi}{2}, -1\right)$ $\left(\frac{\pi}{2}, 1\right)$ $\left(\frac{3\pi}{2}, -1\right)$

d) Complete the table for $x \in \Re$.

Function	Domain	Range
$y = \sin x$	$x \in R$	$-1 \le y \le 1$, $y \in R$
$y = \csc x$	$x \ne n\pi$, $n \in I$	$y \le -1$, $y \ge 1$, $y \in R$

Class Ex. #2

Use a graphing calculator to:

a) Graph $y = \csc x$ and $y = \csc 2x$. $= \left(\frac{1}{\sin(x)}\right)$ $= \left(\frac{1}{\sin(2x)}\right)$

b) State an appropriate graphing calculator window where x is in radians.

c) Complete the table for $x \in \Re$.

Function	Domain	Range	Period	Equation of Asymptotes
$y = \csc x$	$x \ne n\pi$, $n \in I$	$y \le -1$; $y \ge 1$	2π	$x = n\pi$, $n \in I$
$y = \csc 2x$	$x \ne \frac{n\pi}{2}$, $n \in I$	$y \le -1$; $y \ge 2$	$\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$	$x = \frac{n\pi}{2}$, $n \in I$

d) Complete the following statements based on your observations in a), b), c).

i) The graph of $y = \csc 2x$ is a transformation of the graph of $y = \csc x$ by a horizontal compression by a factor of $\frac{1}{2}$ about the y - axis.

ii) Compared to the asymptotes of $y = \csc x$, the asymptotes of the graph of $y = \csc 2x$ are twice as frequent.

Warm-Up #2 — *Review of Absolute Value Functions*

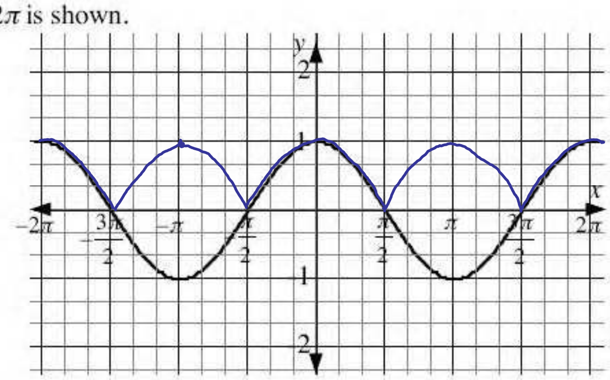
Recall the properties of absolute value functions by completing the following:

- When $f(x) \geq 0$, (i.e. the graph of $y = f(x)$ is above the x-axis), the graph of $y = |f(x)|$ is identical to the graph of $y = f(x)$.
- When $f(x) \leq 0$, (i.e. the graph of $y = f(x)$ is below the x-axis), the graph of $y = |f(x)|$ is a reflection of the graph of $y = f(x)$ in the x-axis.

Class Ex. #3

The graph of $y = \cos x, -2\pi \leq x \leq 2\pi$ is shown.

a) Sketch the graph of $y = |\cos x|$



b) State the domain and range of $y = |\cos x|$.

D: $\{x \mid x \in \mathbb{R}\}$

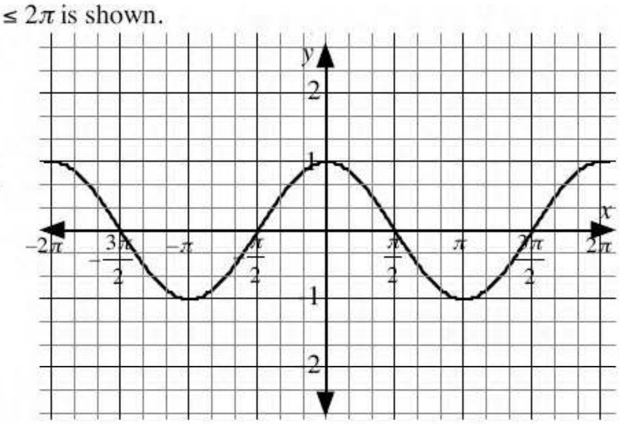
R: $\{y \mid 0 \leq y \leq 1, y \in \mathbb{R}\}$

Complete Assignment Questions #1 - #12

Assignment

1. The graph of $y = \cos x$, $-2\pi \leq x \leq 2\pi$ is shown.

a) Graph $y = \sec x$, the reciprocal of $y = \cos x$.



b) State where $\sec x$ is undefined.

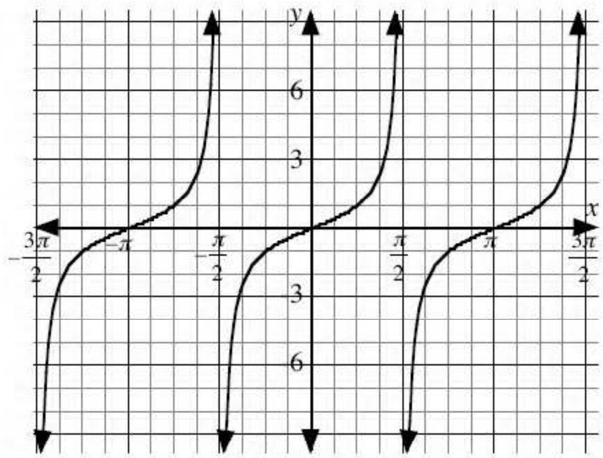
c) List the invariant points.

d) Complete the table for $x \in \Re$.

Function	Domain	Range
$y = \cos x$		
$y = \sec x$		

2. The graph of $y = \tan x$, $-\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$ is shown.

a) Graph $y = \cot x$, the reciprocal of $y = \tan x$.



b) State the equations of the asymptotes of $y = \cot x$.

c) List the invariant points.

d) Complete the table for $x \in \Re$.

Function	Domain	Range
$y = \tan x$		
$y = \cot x$		

3. Use a graphing calculator to:

a) Graph $y = \csc x$ and $y = \csc\left(x + \frac{\pi}{4}\right)$

b) State an appropriate graphing calculator window in radians.

c) Complete the table for $x \in \Re$.

Function	Domain	Range	Period	Equation of Asymptotes
$y = \csc x$				
$y = \csc\left(x + \frac{\pi}{4}\right)$				

4. Use a graphing calculator to:

a) Graph $y = \sec x$ and $y = \sec 3x$

b) State an appropriate graphing calculator window in radians.

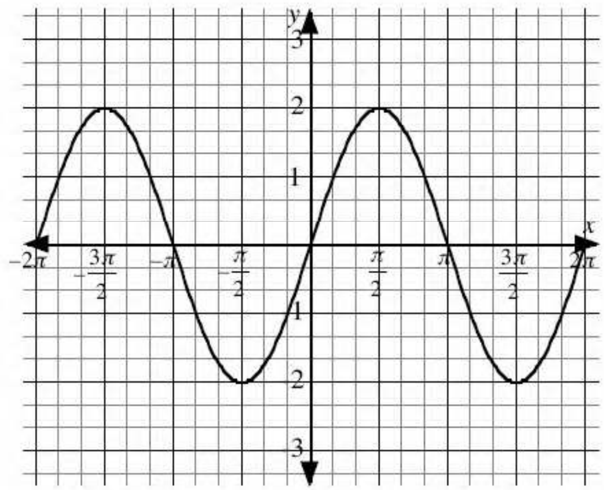
c) Complete the table for $x \in \Re$.

Function	Domain	Range	Period	Equation of Asymptotes
$y = \sec x$				
$y = \sec 3x$				

5. The graph of $y = 2\sin x$, $-2\pi \le x \le 2\pi$ is shown.

a) Graph the reciprocal of $y = 2\sin x$.

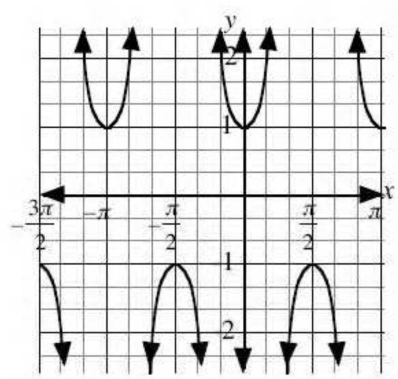
b) State the equation of the reciprocal function.



6. The graph of the function $y = \sec 2x$ is shown.

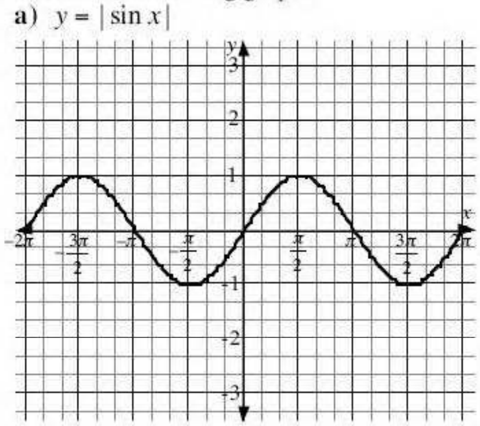
a) Graph the reciprocal of $y = \sec 2x$.

b) State the equation of the reciprocal function.

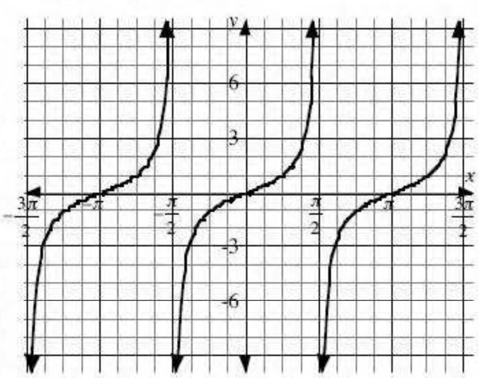


7. Sketch the following graphs

a) $y = |\sin x|$

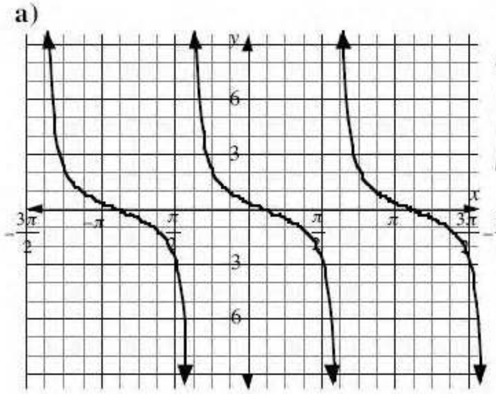


b) $y = |\tan x|$

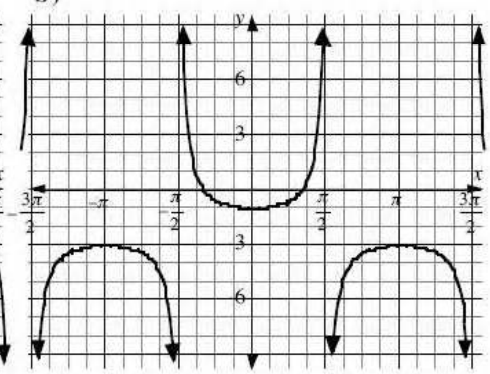


8. The graph represents a reciprocal trigonometric function after a single transformation. Determine the equation of each graph. Verify with a graphing calculator.

a)



b)



Multiple Choice

9. Which of the following describes the asymptotes for $y = \sec x$?

A. $x = n\pi,\ n \in I$

B. $x = \dfrac{\pi}{2} + n\pi,\ n \in I$

C. $x = 2n\pi,\ n \in I$

D. $x = \dfrac{\pi}{2} + 2n\pi,\ n \in I$

10. The minimum positive value of y on the graph of $y = \csc \dfrac{1}{2}x$ is

A. $\dfrac{1}{2}$

B. 1

C. 2

D. impossible to determine

11. The graph of $y = \sec 2x$ is a transformation of the graph of $y = \csc 2x$ by a horizontal translation of

A. $\frac{\pi}{2}$ radians left **B.** $\frac{\pi}{2}$ radians right **C.** $\frac{\pi}{4}$ radians left **D.** $\frac{\pi}{4}$ radians right

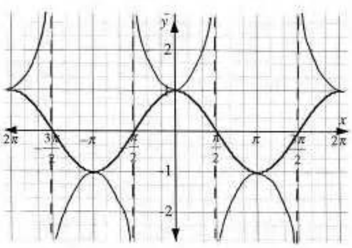
Numerical Response

12. The graph of $y = 4\sin x$ and its reciprocal are drawn. If the reciprocal graph has equation $y = k\csc x$, then the value of k to the nearest hundredth is _____ .

13. The maximum value, to the nearest tenth, of the function $f(x) = |\cos x - 2|$ is _____ .

Answer Key

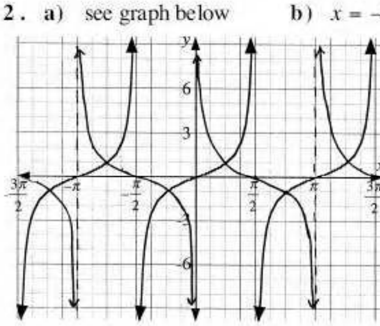
1. a) see graph below **b)** $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ **c)** $(-2\pi, 1), (\pi, -1), (0,1), (\pi, -1), (2\pi, 1)$



d) see table below

Function	Domain	Range
$y = \cos x$	$x \in \Re$	$-1 \le y \le 1, y \in \Re$
$y = \sec x$	$x \ne \frac{\pi}{2} + n\pi, n \in I, x \in \Re$	$y \le -1$ and $y \ge 1, y \in \Re$ or $\lvert y \rvert \ge 1, y \in \Re$

2. a) see graph below **b)** $x = -\pi,\ x = 0,\ x = \pi$



c) $\left(-\frac{5\pi}{4}, -1\right), \left(-\frac{3\pi}{4}, 1\right), \left(-\frac{\pi}{4}, -1\right), \left(\frac{\pi}{4}, 1\right), \left(\frac{3\pi}{4}, -1\right), \left(\frac{5\pi}{4}, 1\right)$

d) See table below

Function	Domain	Range
$y = \tan x$	$x \ne \frac{\pi}{2} + n\pi, n \in I, x \in \Re$	$y \in \Re$
$y = \cot x$	$x \ne n\pi, n \in I, x \in \Re$	$y \in \Re$

3. b) answers may vary

c)

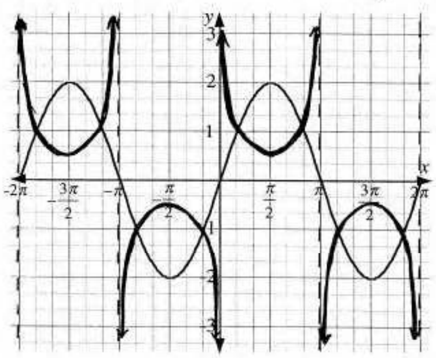
Function	Domain	Range	Per	Asymptotes
$y = \csc x$	$x \ne n\pi, n \in I, x \in \Re$	$y \le -1$ and $y \ge 1, y \in \Re$	2π	$x = n\pi, n \in I$
$y = \csc\left(x + \frac{\pi}{4}\right)$	$x \ne n\pi - \frac{\pi}{4}, n \in I, x \in \Re$	$y \le -1$ and $y \ge 1, y \in \Re$	2π	$x = n\pi - \frac{\pi}{4}, n \in I$

4. b) answers may vary

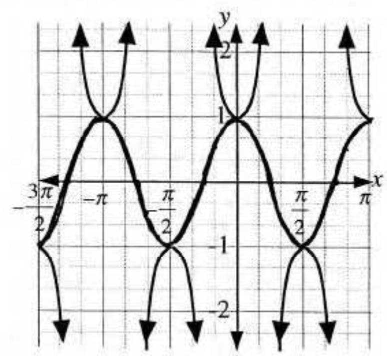
c)

Function	Domain	Range	Per	Asymptotes
$y = \sec x$	$x \neq \frac{\pi}{2} + n\pi, n \in I, x \in \Re$	$y \leq -1$ and $y \geq 1, y \in \Re$	2π	$x = \frac{\pi}{2} + n\pi, n \in I$
$y = \sec 3x$	$x \neq \frac{\pi}{6} + n\frac{\pi}{3}, n \in I, x \in \Re$	$y \leq -1$ and $y \geq 1, y \in \Re$	$\frac{2\pi}{3}$	$x = \frac{\pi}{6} + n\frac{\pi}{3}, n \in I$

5. a) see graph below **b)** $y = \frac{1}{2} \csc x$



6. a) see graph below **b)** $y = \cos 2x$



7. a) see graph below **b)** see graph below

8. a) $y = \cot\left(x + \frac{3\pi}{8}\right)$ **b)** $y = \sec x - 2$

9. B **10.** B **11.** C **12.** 0.25 **13.** 3.0