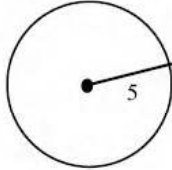# Trigonometry - Functions and Graphs Lesson #4:
Radian Measure

Warm-Up #1

i) What is the formula for the circumference of a circle of radius r? $C = 2\pi r$

ii) Find the circumference of the following circle. Leave your answer as an exact value in terms of π.

$$C = 2\pi r$$
$$= 2\pi (5)$$
$$= 10\pi$$

Warm-Up #2

In all previous work with angular measure we have used **degree** measure .

One degree is defined as $\frac{1}{360}$ of a revolution.

In order to simplify some of the calculations involved in trigonometry and calculus, mathematicians use an alternative angular measure - **radian** measure.
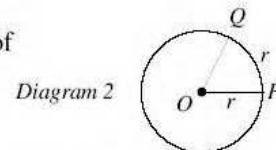
The Radian Measure of an Angle

The radian measure of an angle is a <u>ratio</u> that compares the length of an arc of a circle to the radius of the circle, i.e.

$$\text{measure of an angle in radians} = \frac{\textbf{length of arc subtending the angle}}{\textbf{length of radius}}$$

- The radian measure of $\angle AOB$ is given by the ratio $\dfrac{\text{arc } AB}{\text{radius } OA}$ (see diagram 1)
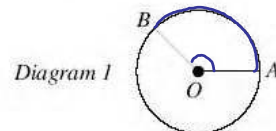
  Diagram 1

- <u>One radian</u> is the measure of the angle at the centre of a circle subtended by an arc equal in length to the radius of the circle (see diagram 2)

  Diagram 2

  $\angle POQ = 1$ **radian**

- Use diagram 1 and the definition to <u>estimate</u> the radian measure of $\angle AOB$.

  about 2 radians

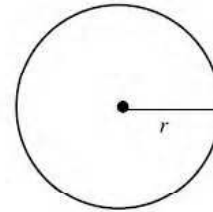- Use diagram 2 to <u>estimate</u> the degree measure of $\angle POQ$.

  about 60°

Converting Between Degrees and Radians

Since an angle can be measured in degrees or radians, it is important in to be able to convert from one measure to the other.

Warm-Up #3

Consider a circle with a radius of r units. Complete the following:

a) i) one complete rotation in degrees is ___360°___ .

ii) the arc length for one complete rotation is _$2\pi r$ or πd_ which is the ___circumference___ of the circle.

iii) the radian measure of an angle of 360° is __$\frac{2\pi r}{r} = 2\pi$__

b) i) one-half rotation in degrees is ___180°___ .

ii) the arc length for one-half rotation is _πr_ .

iii) the radian measure of an angle of 180° is __$\frac{\pi r}{r} = \pi$__.

• In mathematics, the symbol " ° " following a number means the unit of angular measure is degrees.

• If there is no unit after the number, or there is the abbreviation "rad", or the word radians, then the unit is radians.

• For example, if you wish to write the sine ratio for a right angle, you must write sin 90°, and NOT sin 90.

Class Ex. #1

a) Complete the chart:

Degrees	360°	180°	90°	60°	45°	30°	1°
Radians	2π	π	$\frac{2\pi}{4}=\frac{\pi}{2}$	$\frac{2\pi}{6}=\frac{\pi}{3}$	$\frac{2\pi}{8}=\frac{\pi}{4}$	$\frac{\pi}{12}=\frac{\pi}{6}$	$\frac{2\pi}{360}=\frac{\pi}{180}$

b) We note that the rule to convert from **degrees to radians** is to multiply the angle in degrees by

$$\frac{\pi}{180}$$

Class Ex. #2

Convert from degrees to radians (give your answer as an exact value in terms of π)

a) $270° \times \dfrac{\pi}{180} = \dfrac{\overset{3}{270}\pi}{\underset{2}{180}} = \dfrac{3\pi}{2}$

b) $315° \times \dfrac{\pi}{180} = \dfrac{\overset{7}{315}\pi}{\underset{4}{180}} = \dfrac{7\pi}{4}$

Class Ex. #3 Convert the following from degrees to radians (to the nearest tenth)

a) $70° \times \dfrac{\pi}{180}$ = 1.2 rad

b) $205° \times \dfrac{\pi}{180}$ = 3.6 rad

(handwritten diagram of axes:
90 = π/2 = 1.57
180 π = 3.14
0   360° = 2π   6.28
270 = 5π/2 = 4.71)
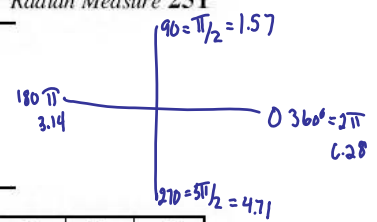
Class Ex. #4 a) Complete the chart:

Radians	2π	π	$\dfrac{\pi}{2}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{4}$	$\dfrac{\pi}{6}$	$\dfrac{\pi}{180}$
Degrees	360°	180°	90°	60°	45°	30°	1°

b) We note that the rule to convert from **radians to degrees** is to multiply the angle in degrees by _____

Class Ex. #5 Convert the following from radians to degrees.

a) $\dfrac{\pi}{4} \times \dfrac{180°}{\pi} = \dfrac{180°}{4} = 45°$

b) $\dfrac{-7\pi}{3} \times \dfrac{\overset{60°}{180}}{\pi} = -420°$

Class Ex. #6 Convert the following radians to degrees (to the nearest tenth)

a) 1.57 radians $1.57 \times \dfrac{180}{\pi} = 90°$

b) 3.2 $\times \dfrac{180}{\pi} = 183.3°$

c) -1.4 rad $\times \dfrac{180}{\pi} = -80.2°$

⁂ **Conversion Chart**

Degrees to Radians   multiply by $\dfrac{\boxed{\pi}}{180}$

Radians to Degrees   multiply by $\dfrac{\boxed{180}}{\pi}$ ⁂

Complete Assignment Questions #1 - #5

Class Ex. #7 a) Use a calculator in **degree mode** to find the value (to 4 decimal places where necessary).

i) sin 45° = 0.7071

ii) sec 135° = $\dfrac{1}{\cos 135°}$ = -1.4142

b) Use a calculator in **radian mode** to find the value (to 4 decimal places where necessary).

i) $\sin \dfrac{\pi}{4}$ = 0.7071

ii) $\sec \dfrac{3\pi}{4} = \dfrac{1}{\cos \frac{3\pi}{4}}$ = -1.4142

c) Find the value (to 4 decimal places) of:

i) $\sin \dfrac{5\pi}{3}$ = -0.8660

ii) cot 135° $\dfrac{1}{\tan 135°}$ = -1.0000
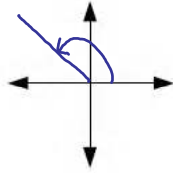
iii) $\tan\left(-\dfrac{5\pi}{6}\right)$ = 0.5774

iv) cos 45 = 0.5253
  No degree sign ∴ radian

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Class Ex. #8

In each of the following :

i) Draw the angle θ in standard position

ii) State the principal angle

iii) Find one positive and one negative coterminal angle for the angle θ

$\frac{\pi}{2} = 90°$

$\frac{\pi}{3} = 60°$

$\frac{\pi}{4} = 45°$

$\frac{\pi}{6} = 30°$

a) $\theta = \dfrac{3\pi}{4}$ $3\frac{\pi}{4} = 3 \times 45 = 135$

b) $\theta = -\dfrac{\pi}{3}$ $= -60°$

principal angle $\dfrac{3\pi}{4}$

principal $\angle = 2\pi - \dfrac{\pi}{3} = \dfrac{6\pi}{3} - \dfrac{\pi}{3} = \dfrac{5\pi}{3}$

$\cot\theta_1 = \dfrac{3\pi}{4} + 2\pi = \dfrac{3\pi}{4} + \dfrac{8\pi}{4} = \dfrac{11\pi}{4}$

$\cot\theta_1 = -\dfrac{\pi}{3} + 2\pi = -\dfrac{\pi}{3} + \dfrac{6\pi}{3} = \dfrac{5\pi}{3}$
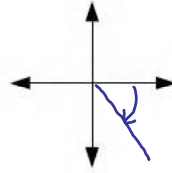
$\cot\theta_2 = \dfrac{3\pi}{4} - 2\pi = \dfrac{3\pi}{4} - \dfrac{8\pi}{4} = -\dfrac{5\pi}{4}$

$\cot\theta_2 = -\dfrac{\pi}{3} - 2\pi = -\dfrac{\pi}{3} - \dfrac{6\pi}{3} = -\dfrac{7\pi}{3}$
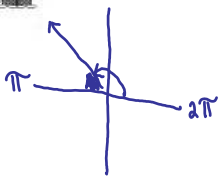

Class Ex. #9

Find the reference angle for the following rotation angles.

a) $\dfrac{5\pi}{6}$ $= 5(30) = 150$

b) $-\dfrac{5\pi}{4}$ $-5(45) = -225$

c) $\dfrac{11\pi}{3}$ $11 \times 60° = 660°$

$\pi - \dfrac{5\pi}{6}$

$\dfrac{6\pi}{6} - \dfrac{5\pi}{6} = \dfrac{\pi}{6}$
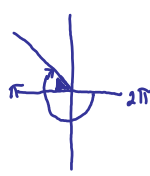
$\dfrac{5\pi}{4} - \pi$

$\dfrac{5\pi}{4} - \dfrac{4\pi}{4}$

$\dfrac{\pi}{4}$
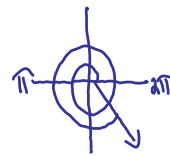
$\dfrac{11\pi}{3} - 2\pi$

$\dfrac{11\pi}{3} - \dfrac{6\pi}{3} = \dfrac{5\pi}{3}$

$2\pi - \dfrac{5\pi}{3} = \dfrac{6\pi}{3} - \dfrac{5\pi}{3} = \dfrac{\pi}{3}$

Complete Assignment Question #6 - #11

Assignment

1. Convert from degrees to radians. Express your answer as an exact value in terms of π.
   a) 30°   **b)** 45°   **c)** 60°   **d)** 135°   **e)** 240°

   f) 150°   **g)** 90°   **h)** 270°   **i)** 225°   **j)** 420°

2. Convert from radians to degrees.
   a) $\dfrac{\pi}{2}$ **b)** $\dfrac{\pi}{4}$ **c)** $-\dfrac{2\pi}{3}$ **d)** $\dfrac{\pi}{6}$

   e) $\dfrac{3\pi}{4}$ **f)** $-\dfrac{3\pi}{2}$ **g)** $\dfrac{7\pi}{4}$ **h)** $-\dfrac{5\pi}{6}$

3. Convert from degrees to radians. Give the answers to 1 decimal place.

   a) 50°   **b)** 205°   **c)** 57.3°   **d)** 250°   **e)** $\left(\dfrac{120}{\pi}\right)^{\circ}$

4. Convert from radians to degrees. Give the answers to the nearest tenth.

   a) 0.5 radians   **b)** 3.1 rad   **c)** 0.4   **d)** 1.8π radians

5. Complete the chart :

Angle in Degrees	0°	30°	45°	60°	90°	120°	135°
Angle in Radians							

Angle in Degrees	150°	180°	210°	225°	240°	270°
Angle in Radians						

Angle in Degrees	300°	315°	330°	360°	540°	720°
Angle in Radians						

6. Find the value (to 4 decimal places where necessary) of

a) $\tan \dfrac{\pi}{4}$ 　　　 b) $\sin 300°$ 　　　 c) $\cos \dfrac{3\pi}{4}$ 　　　 d) $\sin\left(-\dfrac{2\pi}{3}\right)$

e) $\cot 30°$ 　　　 f) $\cos \dfrac{5\pi}{2}$ 　　　 g) $\csc 60$ 　　　 h) $\sec\left(-\dfrac{7\pi}{6}\right)$

7. In each of the following :

i) Draw the angle θ in standard position
ii) State the principal angle
iii) Find one positive and one negative coterminal angle for the angle θ

a) $\theta = \dfrac{5\pi}{4}$ 　　　 b) $\theta = \dfrac{11\pi}{6}$ 　　　 c) $\theta = -\dfrac{2\pi}{3}$ 　　　 d) $\theta = \dfrac{14\pi}{3}$

8. Find the reference angle for the following rotation angles.

a) $\dfrac{7\pi}{6}$ 　　　 b) $\dfrac{3\pi}{4}$ 　　　 c) $\dfrac{11\pi}{6}$ 　　　 d) $-\dfrac{\pi}{6}$

e) $-\dfrac{11\pi}{6}$ 　　　 f) $-\dfrac{5\pi}{3}$ 　　　 g) 5π 　　　 h) $\dfrac{3\pi}{2}$

9. Determine the rotation angle given the reference angle and the quadrant

Reference Angle	Quadrant	Rotation Angle
$\frac{\pi}{3}$	3	
$\frac{\pi}{8}$	1	
$\frac{\pi}{6}$	4	
$\frac{\pi}{12}$	2	
$\frac{\pi}{2}$	between 3 and 4	

Multiple Choice

10. An angle with radian measure 2.36 has degree measure of

    A.   424.80
    B.   135.22
    C.   67.61
    D.   0.04

Numerical Response

11. Correct to the nearest tenth of a degree, $\frac{3\pi}{8}$ rad is equal to _____ °.

Answer Key

1. a) $\dfrac{\pi}{6}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{3}$ d) $\dfrac{3\pi}{4}$ e) $\dfrac{4\pi}{3}$ f) $\dfrac{5\pi}{6}$

   g) $\dfrac{\pi}{2}$ h) $\dfrac{3\pi}{2}$ i) $\dfrac{5\pi}{4}$ j) $\dfrac{7\pi}{3}$

2. a) 90°   b) 45°   c) –120°   d) 30°   e) 135°   f) –270°   g) 315°   h) –150°

3. a) 0.9   b) 3.6   c) 1.0   d) 4.4   e) 0.7
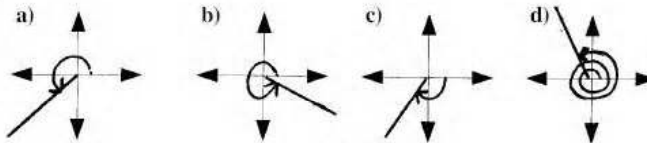
4. a) 28.6°   b) 177.6°   c) 22.9°   d) 324.0°

5.

Angle in Degrees	0°	30°	45°	60°	90°	120°	135°
Angle in Radians	0	$\dfrac{\pi}{6}$	$\dfrac{\pi}{4}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{2}$	$\dfrac{2\pi}{3}$	$\dfrac{3\pi}{4}$

Angle in Degrees	150°	180°	210°	225°	240°	270°
Angle in Radians	$\dfrac{5\pi}{6}$	π	$\dfrac{7\pi}{6}$	$\dfrac{5\pi}{4}$	$\dfrac{4\pi}{3}$	$\dfrac{3\pi}{2}$

Angle in Degrees	300°	315°	330°	360°	540°	720°
Angle in Radians	$\dfrac{5\pi}{3}$	$\dfrac{7\pi}{4}$	$\dfrac{11\pi}{6}$	2π	3π	4π

6. a) 1   b) –0.8660   c) –0.7071   d) –0.8660   e) 1.7321   f) 0   g) –3.2807   h) –1.1547

7.

   a)   b)   c)   d)

(ii) $\dfrac{5\pi}{4}$ $\dfrac{11\pi}{6}$ $\dfrac{4\pi}{3}$ $\dfrac{2\pi}{3}$

(iii) $\dfrac{13\pi}{4}, \dfrac{-3\pi}{4}$ $\dfrac{23\pi}{6}, \dfrac{-\pi}{6}$ $\dfrac{10\pi}{3}, \dfrac{-8\pi}{3}$ $\dfrac{2\pi}{3}, \dfrac{-4\pi}{3}$ Answers may vary

8. a) $\dfrac{\pi}{6}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{6}$ d) $\dfrac{\pi}{6}$ e) $\dfrac{\pi}{6}$ f) $\dfrac{\pi}{3}$ g) 0   h) $\dfrac{\pi}{2}$

9. $\dfrac{4\pi}{3}$ $\dfrac{\pi}{8}$ $\dfrac{11\pi}{6}$ $\dfrac{11\pi}{12}$ $\dfrac{3\pi}{2}$ 10. B   11. 67.5