Trigonometry - Functions and Graphs Lesson 5:
Applications of Radian Measure

Solving Simple Trigonometric Equations in Radian Measure

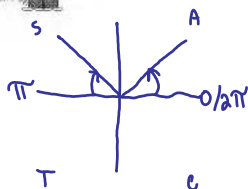**Class Ex. #1**

Find each value of θ for $0 \leq \theta \leq 2\pi$.

a) $\sin \theta = \dfrac{1}{2}$ $\quad$ ref $\theta = 30° = \dfrac{\pi}{6}$

$\theta_1 = 0 + \dfrac{\pi}{6} = \dfrac{\pi}{6}$

$\theta_2 = \pi - \dfrac{\pi}{6}$

$\dfrac{6\pi}{6} - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$

b) $\sec \theta = -\sqrt{2}$

$\cos \theta = \dfrac{1}{\sqrt{2}}$ $\quad$ ref $\theta = 45° = \dfrac{\pi}{4}$

$\theta_1 = \dfrac{4\pi}{4} - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

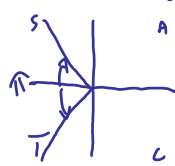$\theta_2 = \dfrac{4\pi}{4} + \dfrac{\pi}{4} = \dfrac{5\pi}{4}$

c) $\sqrt{\tan^2 \theta} = \sqrt{3}$

$\tan \theta = \pm\sqrt{3}$ $\quad$ ref $L = 60° = \dfrac{\pi}{3}$

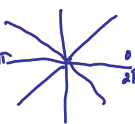$\theta_1 = 0 + \dfrac{\pi}{3} = \dfrac{\pi}{3}$

$\theta_2 = \dfrac{3\pi}{3} - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

$\theta_3 = \dfrac{3\pi}{3} + \dfrac{\pi}{3} = \dfrac{4\pi}{3}$

$\theta_4 = 2\pi - \dfrac{\pi}{3}$

$\dfrac{6\pi}{3} - \dfrac{\pi}{3} = \dfrac{5\pi}{3}$
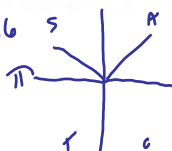

Class Ex. #2

Solve the following equations. Give your answer to the nearest hundredth.

a) $\sin x = 0.425$, $0 \leq x \leq 2\pi$
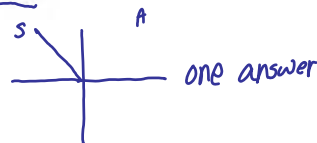
ref $x = 0.43896$

$x_1 = .44$

$x_2 = \pi - .44 = 2.70$

b) $\tan x = -\dfrac{4}{5}$, $0 \leq x \leq \pi$

ref $x = .6747$

one answer

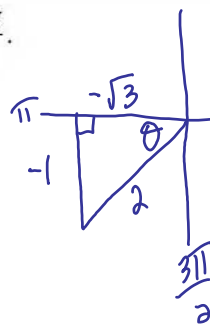$x_1 = \pi - .6747$

$= 2.47$

Class Ex. #3

Find the value of $\csc x$ if $\tan x = \dfrac{1}{\sqrt{3}} \dfrac{o}{a}$ and $\pi \leq x \leq \dfrac{3\pi}{2}$.

$\csc x = \dfrac{h}{o} = \dfrac{2}{-1} = -2$

$(-1)^2 + (-\sqrt{3})^2 = r^2$

$1 + 3 = r^2$

$\sqrt{4} = \sqrt{r^2}$

$r = \pm 2$

Complete Assignment Questions #1 - #5

Arc Length

In the last lesson we defined the radian measure of an angle as

measure of an angle in radians = $\dfrac{\text{length of arc subtending the angle}}{\text{length of radius}}$ i.e. $\theta = \dfrac{a}{r}$

Use the formula below to solve problems involving arc length, radius and central angle.

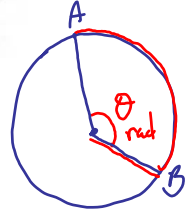$$a = r\theta$$

NOT on the formula sheet

where, θ = the measure of the angle in **radians**

a = length of the arc around the angle

r = length of radius

Class Ex. #4 A pendulum 30 cm long swings through an arc of 45 cm. Through what angle does the pendulum swing. Answer in <u>degrees</u> and in <u>radians</u> to the nearest <u>tenth</u>.

$a = r\theta$

$\theta = \dfrac{a}{r} = \dfrac{45}{30} = 1.5$ radians

$1.5 \times \dfrac{180}{\pi} = 85.9°$

Class Ex. #5 A circle of radius 3 cm contains a central angle of 2.4 radians. Calculate the length of the arc subtended by the central angle, to the nearest tenth of a centimetre.

$a = r\theta = 3(2.4) = 7.2$ cm

Class Ex. #6 Calculate the arc length (to the nearest tenth of a metre) of a sector of a circle with diameter 9.2 m if the sector angle is <u>150°</u>.

$r = 9.2 \div 2$
$\quad = 4.6$

$a = r\theta = 4.6 \times \dfrac{5\pi}{6} = 12.0$ m

$\theta = 150 \times \dfrac{\pi}{180} = \dfrac{5\pi}{6}$

Class Ex. #7 A circle with centre C and minor arc AB measuring 15.2 cm is shown. If $\angle ABC = \angle BAC = \dfrac{\pi}{6}$ radians, find the length of the radius of the circle to the nearest tenth of a centimetre.

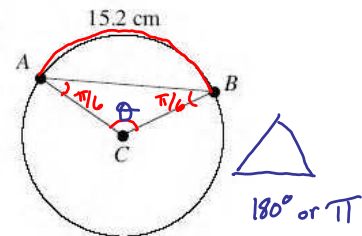$a = r\theta$ $r = \dfrac{15.2}{\dfrac{2\pi}{3}} = 7.3$ cm

$r = \dfrac{a}{\theta}$

$a = 15.2$

$\theta = \pi - \dfrac{2\pi}{6}$

$= \dfrac{5\pi}{3} - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

180° or π

Complete Assignment Questions #6 - #16

Assignment

1. Find each value of θ for $0 < \theta < 2\pi$

 a) $\sin \theta = \dfrac{1}{2}$

 b) $\cos \theta = -\dfrac{1}{\sqrt{2}}$

 c) $\tan \theta = -1$

 d) $2 \sin \theta = \sqrt{2}$

 e) $\cot \theta = -\sqrt{3}$

 f) $2\cos \theta + 1 = 0$

 g) $\csc \theta = 1$

 h) $\sec \theta - 2 = 0$

 i) $\sqrt{3} \cot \theta = 1$

2. Solve the following equations. Give your answer to the nearest hundredth of a radian.

   a) $\tan x = 0.5371,\ 0 \le x \le 2\pi$

   b) $\cos x = -\dfrac{4}{5},\ \pi \le x \le 2\pi$

   c) $\csc x = 6,\ 0 \le x \le \pi$

   d) $\cot x = -1.5,\ 0 \le x \le 2\pi$

3. Find the values of each angle θ if $0 \le \theta \le 2\pi$.

   a) $\sin^2 \theta = \dfrac{1}{2}$

   b) $\tan^2 \theta = \dfrac{1}{3}$

   c) $\sec^2 \theta = \dfrac{4}{3}$

   d) $\csc^3 \theta = -8$

4. If $\tan x = -\sqrt{3}$ and $\dfrac{\pi}{2} < x < \pi$, then find the value of $\csc x$.

5. If $\sec \theta = -\dfrac{2}{\sqrt{3}}$ and $\pi \le \theta \le \dfrac{3\pi}{2}$, what is the value of $\cot \theta$?

6. A circle has radius 8 cm. Determine (in radians) the measure of the central angle subtended by an arc of length 5.6 cm.

7. Calculate the arc length (to the nearest tenth of a metre) of a sector of a circle with radius 8.4 m if the sector angle is 80°.

8. In the diagram the circle with centre C has radius 6 cm. Find the length of arc ADB, to the nearest tenth of a centimetre, if $\angle CAB = \dfrac{\pi}{8}$.
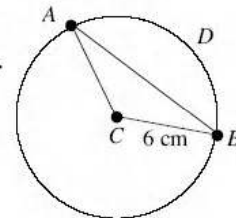
9. An arc of a circle is 1.5 cm long and subtends an angle of 3 radians at the centre of the circle. Calculate the radius of the circle.

10. Two sectors of the same circle have sector angles of 30° and 105° respectively. The arc length of the smaller sector is 12 cm.

    a) What is the radius of the circle ?

    b) What is the arc length of the larger sector?

11. A pendulum swings through an angle of 45°. Find the length of the pendulum (to the nearest cm) if the end of the pendulum swings through an arc of length 32 cm.

12. If $\tan X = -2$ and $\dfrac{3\pi}{2} \le X \le 2\pi$, the value of csc X is

    A. $-\sqrt{5}$

    B. $\sqrt{5}$

    C. $-\dfrac{\sqrt{5}}{2}$

    D. $\dfrac{\sqrt{5}}{2}$

13. An arc of a circle subtends a central angle $x°$. If the length of the arc is 1.2 cm and the diameter of the circle is 4 cm, then the value of x to the nearest whole number is

A. 17

B. 34

C. 54

D. 108

Numerical Response

14. An arc DE of a circle, center O, is $\frac{1}{6}$ of the circumference. The size of $\angle DOE$, to the nearest one hundredth of a radian, is _____ .

15. A person on a Ferris wheel moves a distance of 5 metres from position P to position Q. If the diameter of the wheel is 18 metres, the measure of the central angle, to the nearest tenth of a degree, is _____ .

16. A satellite makes one complete revolution of the earth in 90 min. Assume that the orbit is circular and that the satellite is situated 280 km above the equator. If the radius of the earth at the equator is 6400 km, then the speed of the satellite, in kilometres per second, to the nearest one hundredth, is _____ .

Answer Key

1. a) $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$ b) $\dfrac{3\pi}{4}, \dfrac{5\pi}{4}$ c) $\dfrac{3\pi}{4}, \dfrac{7\pi}{4}$ d) $\dfrac{\pi}{4}, \dfrac{3\pi}{4}$ e) $\dfrac{5\pi}{6}, \dfrac{11\pi}{6}$

   f) $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}$ g) $\dfrac{\pi}{2}$ h) $\dfrac{\pi}{3}, \dfrac{5\pi}{3}$ i) $\dfrac{\pi}{3}, \dfrac{4\pi}{3}$

2. a) 0.49, 3.63 b) 3.79 c) 0.17, 2.97 d) 2.55, 5.70

3. a) $\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$ b) $\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$ c) $\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$ d) $\dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

4. $\dfrac{2\sqrt{3}}{3}$ 5. $\sqrt{3}$ 6. 0.7 rad 7. 11.7 m 8. 14.1

9. 0.5 cm 10. a) 22.9 cm b) 42 cm 11. 41 cm

12. C 13. B 14. 1.05 15. 31.8° 16. 7.77