Trigonometry - Functions and Graphs Lesson 7:
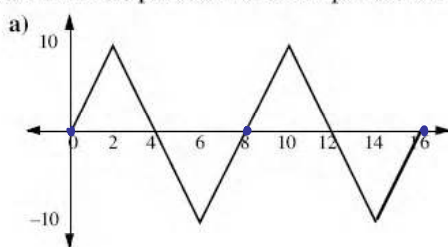## Graphing Primary Trigonometric Functions

Periodic Functions

A **periodic function** is a function whose graph repeats regularly over some interval of the domain. The length of this interval is called the **period** of the function.

The **amplitude** of a periodic function is defined as half the distance between the maximum and minimum values of the function.
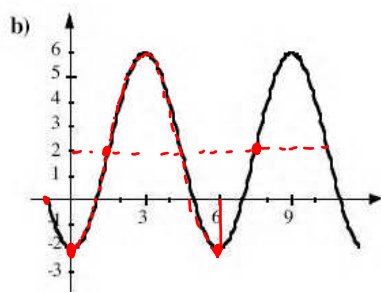

Class Ex. #1

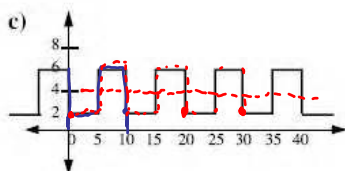Determine the period and the amplitude for each of the following periodic functions.

a)

period = 8 units

amplitude = $\dfrac{10-(-10)}{2} = \dfrac{20}{2} = 10$ units

b)

period = 6 units

amplitude = $\dfrac{6-(-2)}{2} = \dfrac{8}{2} = 4$ units

c)

period = 10 units

amplitude = $\dfrac{6-2}{2} = \dfrac{4}{2} = 2$ units

Graphing the Primary Trigonometric Functions

In this lesson we will learn the graphs of $y = \sin x$, $y = \cos x$, and $y = \tan x$.

To investigate the graphs of trigonometric functions we will use the table of values method to graph each function on a domain of $0° \le x \le 360°$ and use the graphing calculator to complete the graph.

We will develop the graph of $y = \sin x$ as a class lesson and the graphs of $y = \cos x$ and $y = \tan x$ will be assignment questions.

| Exploration | *Graphing $y = \sin x$ where x is in Degrees* |

a) Complete the following table of values for domain $0° \le x \le 360°$. Give your answers to two decimal places where necessary.

x	0°	30°	60°	90°	120°	150°	180°
$y = \sin x$	0	0.5	0.87	1	0.87	0.5	0

x	210°	240°	270°	300°	330°	360°
$y = \sin x$	−0.5	−0.87	−1	−0.87	−0.5	0

b) Plot the points on the grid below. Do not join the points.

c) Graph $y = \sin x$ on your calculator using degree mode and the following window format.
$$x: [-360, 540, 30]$$
$$y: [-1.2, 1.2, 0.2]$$

d) On the grid, copy the graph from c) to complete the graph of $y = \sin x$, $-360° \le x \le 540°$.

Note
- $y = \sin x$ is a periodic function whose graph continues indefinitely to the left and to the right.

- The graph shown in b) is a partial graph of $y = \sin x$ on a restricted domain.

Class Ex. #2
State the following for the function $y = \sin x$ where x is in degree measure.

a) Domain ___$x \in R$___

b) Range ___$\{y \mid -1 \le y \le 1, y \in R\}$___

c) Amplitude ___1___

d) Period ___360°___

e) x-intercept(s)___$0°, 180°, 360°$ . . . . . ___ $180°n, n \in I$

f) y-intercept(s)___0___

Class Ex. #3 A student was asked to reproduce the graph in Exploration b) in radian mode.

a) Write down the graphing calculator window format the student should use.

$x: \left[-2\pi, 3\pi, \dfrac{\pi}{6}\right]$ $y: \left[-1.2, 1.2, .2\right]$

b) Sketch the graph of $y = \sin x$ for $0 \le x \le 2\pi$ showing the intercepts.
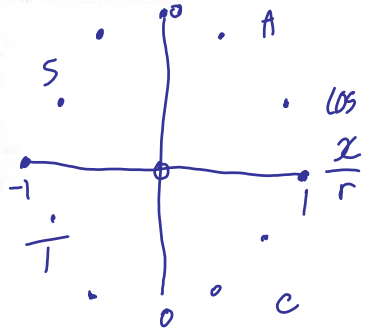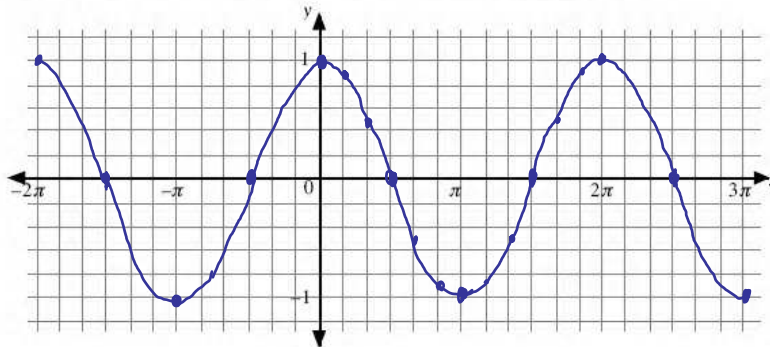
$y = \sin x$

Complete Assignment Questions #1 - #12

Assignment

1. a) Complete the following table of values for domain $0 \le x \le 2\pi$. Give your answers to two decimal places where necessary.

x	0	$\dfrac{\pi}{6}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{2}$	$\dfrac{2\pi}{3}$	$\dfrac{5\pi}{6}$	π
$y = \cos x$	1	0.87	0.5	0	-0.5	-0.87	-1

x	$\dfrac{7\pi}{6}$	$\dfrac{4\pi}{3}$	$\dfrac{3\pi}{2}$	$\dfrac{5\pi}{3}$	$\dfrac{11\pi}{6}$	2π
$y = \cos x$	-0.87	-0.5	0	0.5	0.87	1

b) Plot the points on the grid below. Do not join the points.

c) Graph $y = \cos x$ on your calculator using radian mode and the following window format.

$$x: \left|-2\pi, 3\pi, \dfrac{\pi}{6}\right|$$
$$y: \left[-1.2, 1.2, 0.2\right]$$

d) On the grid, copy the graph from c) to complete the graph of $y = \cos x$, $-2\pi \le x \le 3\pi$.

2. State the following for the function $y = \cos x$ where x is in radian measure.

 a) Domain ___$x \in R$___ **b)** Range ___$\{y \mid -1 \le y \le 1\}$___

 c) Amplitude ___1___ **d)** Period ___2π or $360°$___

 e) x-intercept(s) $\frac{\pi}{2}, \frac{3\pi}{2} \ldots$ etc **f)** y-intercept(s)___1___
 $\frac{\pi}{2} + n\pi, n \in I$

3. A student was asked to reproduce one complete cycle of the graph of $y = \cos x$ starting from zero degrees.

 a) Write down a graphing calculator window format the student could use.

 $x = [0, 360, 30]$ $y = [-1.2, 1.2, .2]$

 b) Sketch the graph of $y = \cos x$, $0° \le x \le 360°$, showing the intercepts.

 $y = \sin x$
 $y = \cos x$

4. Complete the following:
 a) When $\sin x$ has a maximum value, the value of $\cos x$ is ___0___.

 b) When $\sin x$ has a minimum value, the value of $\cos x$ is ___0___.

 c) When $\sin x$ has a value of zero, the value of $\cos x$ is ___± 1___.

5. **a)** Using the same grid sketch the graph of $y = \sin x$ and $y = \cos x$ for domain $-\pi \le x \le 2\pi$.

 b) For what values of x, in the domain $0 \le x \le 2\pi$, do $\sin x$ and $\cos x$ have the same value?

 $\frac{\pi}{4}, \frac{5\pi}{4}$

 c) What is the minimum horizontal translation applied to the graph of $y = \cos x$ which would result in the graph of $y = \sin x$? $\frac{\pi}{2}$ units to the right

 d) If $\sin x = \cos(x - c)$ for $x \in \Re$, find the smallest positive value of c.

 $\sin x = \cos\left(x - \frac{\pi}{2}\right)$ $c = \frac{\pi}{2}$

 e) If $\cos x = \sin(x - k)$ for $x \in \Re$, find a value for k.

 $\cos x = \sin\left(x - \left(-\frac{\pi}{2}\right)\right)$ $k = -\frac{\pi}{2}$ or $+\frac{3\pi}{2}$

6. **a)** Complete the following table of values for domain $0° \leq x \leq 180°$. Give your answers to two decimal places where necessary.

x	0°	15°	30°	45°	60°	75°	90°	Asymtote
$y = \tan x$	0	0.27	0.58	1	1.73	3.73	undefined	

x	105°	120°	135°	150°	165°	180°
$y = \tan x$	-3.73	~1.73	~1	~0.58	-0.27	0

b) Plot the points on the grid below. Do not join the points.
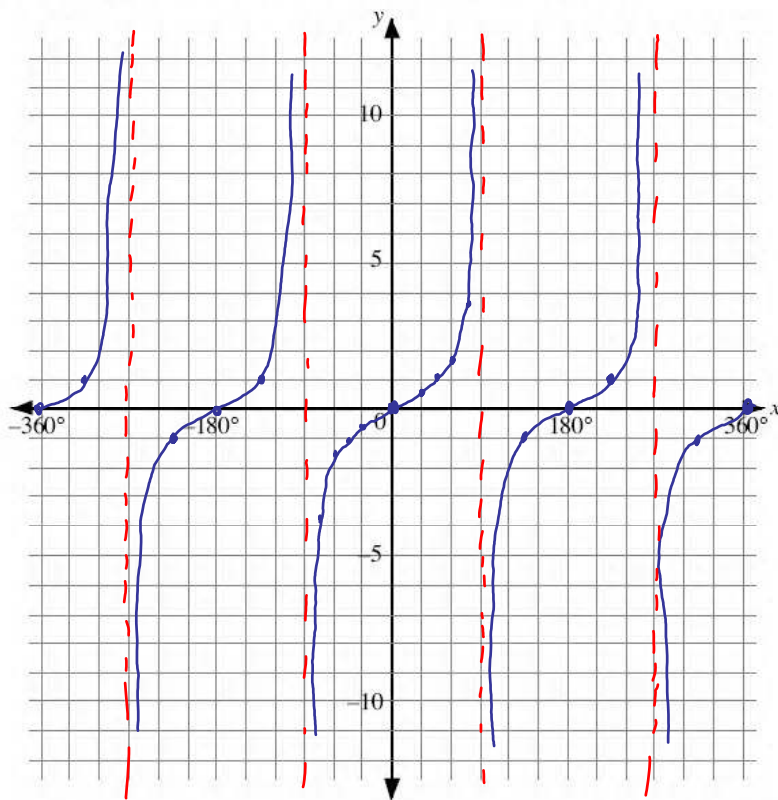
c) To investigate what happens to the graph of $y = \tan x$ as x approaches 90°, complete the following table.

x	80°	85°	89°	89.9°	89.99°	89.999°
$y = \tan x$	5.67	11.43	57.3	573	5730	57 296

x	100°	95°	91°	90.1°	90.01°	90.001°
$y = \tan x$	~5.67	~11.43	~57.3	-573	-5731	-57296

d) To graph $y = \tan x$ on your calculator using degree mode use the following instructions:

   Step 1: Enter $y = \tan x$ using the [Y=] key.

   Step 2: Press the [Zoom] key.

   Step 3: Access "ZTrig" and press [Enter].

e) Write down the graphing calculator window format for "ZTrig".

   $x : [-360, 360, 90]$ $y [-4, 4, 1]$

f) On the grid in b) complete the graph of $y = \tan x$ for domain $-360° \le x \le 360°$.

g) Is $y = \tan x$ a periodic function? If so what is the period?   yes period $180°$

h) Does the concept of amplitude apply to the graph of $y = \tan x$?   No

7. State the following for the function $y = \tan x$, $0° \le x \le 360°$.

   a) Domain $\{x \mid x \ne 90°, 270°\}$ b) Range $y \in \mathbb{R}$

   c) Period $180°$ d) x-intercept(s) $0°, 180°, 360°$

   e) y-intercept(s) 0

   f) Equations of vertical asymptotes $x = 90°$ $x = 270°$

Multiple Choice   Questions #8 - #11 refer to the graph of the function $f(x) = \tan x$, $x \in \mathbb{R}$

8. The domain of the function is

   A. $x \ne n\pi, \ n \in I$
   B. $x \ne \dfrac{n\pi}{2}, \ n \in I$
   C. $x \ne \dfrac{\pi}{2} + n\pi, \ n \in I$ ⟸⟹
   D. $x \in \mathbb{R}$

9. The asymptotes have equations

   A. $x = n\pi, \ n \in I$
   B. $x = \dfrac{n\pi}{2}, \ n \in I$
   C. $x = \dfrac{\pi}{2} + n\pi, \ n \in I$
   D. $x = 2n\pi, \ n \in I$

10. The range of the function is

   A. $x \ne \dfrac{\pi}{2} + n\pi, \ n \in I$
   B. $x \in \mathbb{R}$
   C. $f(x) \ne \dfrac{\pi}{2} + n\pi, \ n \in I$
   D. $f(x) \in \mathbb{R}$

11. The x-intercepts of the graph are

   A. $x = n\pi, \ n \in I$
   B. $x = \dfrac{n\pi}{2}, \ n \in I$
   C. $x = \dfrac{\pi}{2} + n\pi, \ n \in I$
   D. $x = 2n\pi, \ n \in I$

12. Which of the following statements is incorrect?

A. The graph of $y = \sin x$ has an x-intercept of π. ✓

B. The graph of $y = \cos x$ has a minimum value when $x = \pi$. ✓

C. The graph of $y = \tan x$ has a an x-intercept of π. ✓

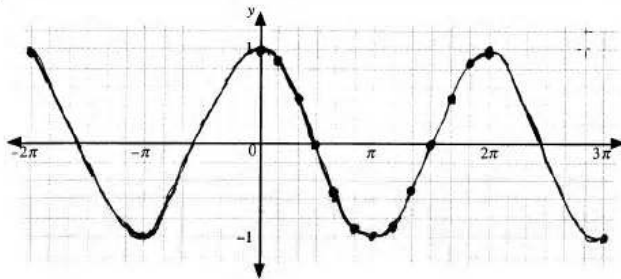**D.** The graph of $y = \cos x$ has an x-intercept of π.

Answer Key

1. a)

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$y = \cos x$	1	0.87	0.5	0	−0.5	−0.87	−1

x	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$y = \cos x$	−0.87	−0.5	0	0.5	0.87	1

d)

2. a) $x \in \Re$ **b)** $\{y \mid -1 \le y \le 1, y \in \Re\}$ **c)** 1   **d)** 2π **e)** $\frac{\pi}{2} + n\pi, n \in I$ **f)** 1

3. a) $x:[0, 360, 30]$ $y:[-1.2, 1.2, 0.2]$ answers may vary   **b)**

4. a) 0   **b)** 0   **c)** ±1

5. a) → → → → → →

b) $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

c) $\frac{\pi}{2}$ units to the rigth

d) $\frac{\pi}{2}$ **e)** $-\frac{\pi}{2}$

6. a)

x	0°	15°	30°	45°	60°	75°	90°
$y = \tan x$	0	0.27	0.58	1	1.73	3.73	undefined

x	105°	120°	135°	150°	165°	180°
$y = \tan x$	−3.73	−1.73	−1	−0.58	−0.27	0

c)

x	80°	85°	89°	89.9°	89.99°	89.999°
$y = \tan x$	5.67	11.43	57.29	573	5730	57 296

x	100°	95°	91°	90.1°	90.01°	90.001°
$y = \tan x$	−5.67	−11.43	−57.29	−573	−5730	−57 296

e) x:[−352.5, 352.5, 90] y:[−4, 4, 1] **f)**
g) Yes, the period is 180°
h) no

7. a) $\{x \mid x \neq 90°, 270°\}$
   b) $y \in \Re$
   c) 180°
   d) 0°, 180°, 360°
   e) 0
   f) $x = 90°, x = 270°$

8. C   **9.** C

10. D   **11.** A

12. D