Trigonometry - Functions and Graphs Lesson #8:
Transformations of Trigonometric Functions Part 1

Warm-Up *Introduction*

In the next two lessons we will consider the graphs of the functions whose equations are

$y = a \sin[b(x - c)] + d$ and $y = a \cos[b(x - c)] + d$

and relate them to the graphs of the functions whose equations are $y = \sin x$ and $y = \cos x$.

In this lesson we concentrate on the effects of the parameters a and b.

a) Use the knowledge gained from the transformation unit to describe how the graph of the given function compares to the graph of $y = \sin x$, where x is in degrees

i) $y = 2 \sin x$ $a=2$ vert exp by factor of 2

ii) $y = \sin 2x$ $b=2$ hor comp by factor of $1/2$

iii) $y = -3 \sin x$ $a=-3$ vert exp by factor of 3, reflection over x axis

iv) $y = \sin(-3x)$ $b=-3$ hor comp by factor of $\frac{1}{3}$, reflection over y axis

b) In which of the above examples is there a change in amplitude compared to the graph of $y = \sin x$? i) and iii)

c) In which of the above examples is there a change in period compared to the graph of $y = \sin x$? ii) and iv)

d) Complete the table. Use a graphing calculator if necessary.

Equation	Amplitude	Period	
$y = \sin x$	1	360°	
$y = 2 \sin x$	2	360°	
$y = \sin 2x$	1	180°	factor $\frac{1}{2}$
$y = -3 \sin x$	3	360°	
$y = \sin(-3x)$	1	120°	factor $\frac{1}{3}$
$y = 5 \sin 4x$	5	90°	factor $\frac{1}{4}$
$y = \frac{1}{3} \sin \frac{1}{2}x$	$\frac{1}{3}$	720°	factor 2
$y = a \sin bx$	a	$\frac{360}{b}$	factor $\frac{1}{b}$

e) Describe the effect of the parameter "a" on the graphs of $y = a \sin x$.

$|a|$ = amplitude

f) Describe the effect of the parameter "b" on the graphs of $y = \sin bx$.

Period = $\frac{360}{|b|}$

g) Would you expect similar effects on the graph of $y = a \cos bx$? Investigate if necessary.

yes

Effects of a and b in y = a sin *bx, y = a* cos *bx, and y = a* tan *bx*

Changing the parameter "*a*" on the graphs of $y = a \sin x$ and $y = a \cos x$ results in a vertical stretch about the *x*-axis with the following:

- an expansion if $a > 1$
- a compression if $0 < a < 1$
- if $a < 0$, the result is a reflection in the *x*-axis and a vertical stretch of factor a about the *x*-axis.

Changing the parameter "*a*" on the graph of $y = a \tan x$ also results in a vertical stretch of factor "*a*" about the *x*-axis.

Changing the parameter "*b*" on the graphs of $y = \sin bx$, $y = \cos bx$, and $y = \tan bx$ results in a horizontal stretch about the *y*-axis with the following:

- a compression if $b > 1$
- an expansion if $0 < b < 1$
- if $b < 0$, the result is a reflection in the *y*-axis and a horizontal stretch of factor $\frac{1}{b}$ about the *y*-axis.

$y = a \sin bx$ or $y = a \cos bx$

amplitude $= |a| = \dfrac{\text{Max} - \text{Min}}{2}$

period $= \dfrac{360°}{|b|}$ (for degree measure)

period $= \dfrac{2\pi}{|b|}$ (for radian measure)

$y = a \tan bx$

amplitude is not applicable

period $= \dfrac{180°}{|b|}$ (for degree measure)

period $= \dfrac{\pi}{|b|}$ (for radian measure)

Hints for Graphing a Trigonometric Function Manually

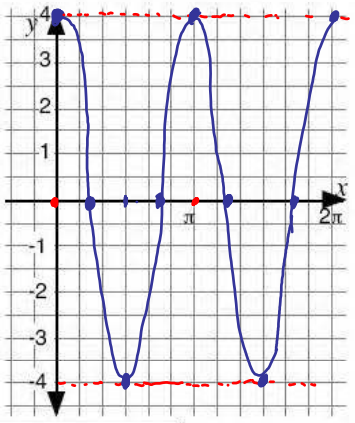
- Sketch the primary trigonometric graph, i.e. $y = \sin x$ or $y = \cos x$
- Adjust the basic graph for any change in amplitude by considering the max and min points.
- Adjust the new graph for any change in period by dividing the period into four parts using the maximum and minimum points, and the points where the graph intersects the mid-line (the horizontal line running through the centre of the graph).

Consider the graph of $y = 4\cos 2x,\ 0 \le x \le 2\pi$.

a) State the amplitude and period.

$|a| = 4$ period $= \frac{2\pi}{2} = \pi$

b) Sketch the graph on the grid. Use a graphing calculator to verify.



Write the equation of:

a) a sine function having an amplitude of $\frac{2}{3}$ and a period of $\frac{\pi}{6}$.

$a = \frac{2}{3}$ $\quad p = \frac{2\pi}{b}$ $\therefore b = \frac{2\pi}{\frac{\pi}{6}} = 2\cancel{\pi} \times \frac{6}{\cancel{\pi}} = 12$ $\quad y = \frac{2}{3}\sin 12x$

b) a cosine function having an amplitude of 3 and a period of 720°.

$a = 3$ $\quad b = \frac{360^\circ}{p} = \frac{360^\circ}{720} = \frac{1}{2}$ $\quad y = 3\cos\frac{1}{2}x$

c) a tangent function having a period of $\frac{\pi}{2}$.

$p = \frac{\pi}{b} \therefore b = \frac{\pi}{p} = \frac{\pi}{\frac{\pi}{2}} = \cancel{\pi} \times \frac{2}{\cancel{\pi}} = 2$ $\quad y = \tan 2x$

Consider the graph shown.

a) State the amplitude and period.

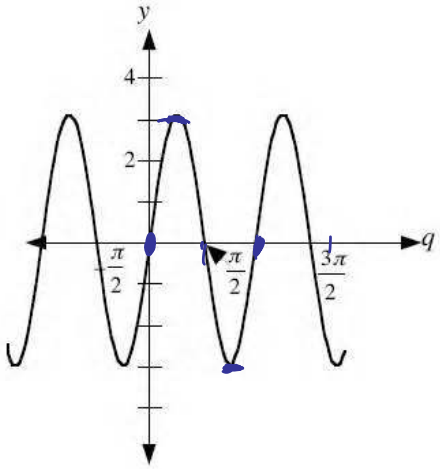
$a = 3$ $\quad$ period $= \pi$

$b = \frac{2\pi}{\pi} = 2$

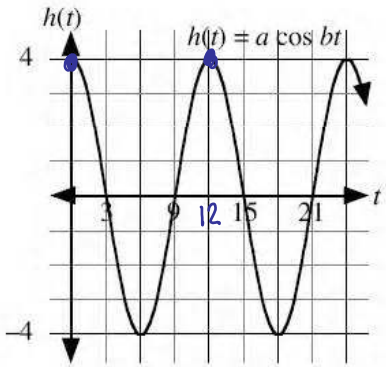
b) Write the equation of the sine function which the graph represents.

$y = a\sin bx$

$y = 3\sin 2x$



The graph represents the effect of tides on mean sea level over a 24 hour period. The graph has equation $h(t) = a \cos bt$, where t is in hours and h is the height, in metres, relative to mean sea level. Determine the equation of the graph.



$a = 4 \quad b = \frac{2\pi}{P} = \frac{2\pi}{12} = \frac{\pi}{6}$

$h(t) = 4 \cos \frac{\pi}{6} t$

Complete Assignment Questions #1 - #16

Assignment

1. Describe how the graph of the given function compares to the graph of $y = \cos x$.

a) $y = 5 \cos x$

b) $y = 2 \cos \frac{1}{2}x$

c) $y = -\frac{1}{3} \cos 4x$

d) $y = 0.2 \cos(-6x)$

2. State the amplitude

a) $y = 5 \sin x$ **b)** $y = \cos 3x$ **c)** $y = \frac{7}{3} \sin 2x$ **d)** $y = -4 \cos \frac{5}{6}\theta$

3. State the period in degrees.

a) $y = 6 \sin x$ **b)** $y = \tan 3x$ **c)** $y = \frac{2}{3} \cos \frac{x}{7}$ **d)** $y = -2 \tan \frac{2}{3}\theta$

4. State the period in radians.

a) $y = 7 \tan x$ **b)** $y = \cos 3x$ **c)** $y = \frac{1}{4} \sin \frac{x}{3}$ **d)** $y = 5 \tan \frac{1}{2}\theta$

5. Write the equation of a **sine** function having the indicated amplitude and period.

a) amplitude 2, period 1080° **b)** amplitude 8, period $\frac{\pi}{4}$ **c)** amplitude $\frac{3}{2}$, period 6π

6. Write the equation of a **cosine** function having the indicated amplitude and period.

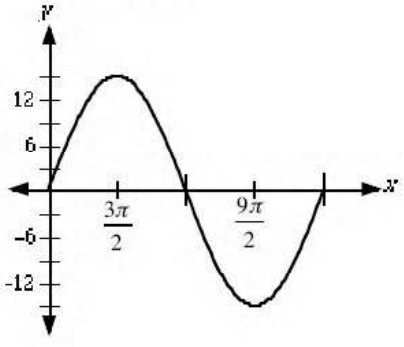
a) amplitude 1, period 180° **b)** amplitude 5, period $\frac{4\pi}{3}$ **c)** amplitude $\frac{5}{3}$, period 3π

7. Write the equation of a **tangent** function having the indicated period.

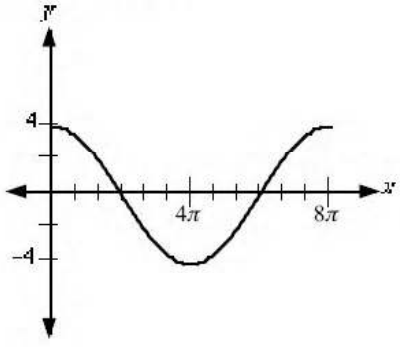
a) period 45° **b)** period $\frac{4\pi}{3}$

8. Determine the equation of each graph in the form:

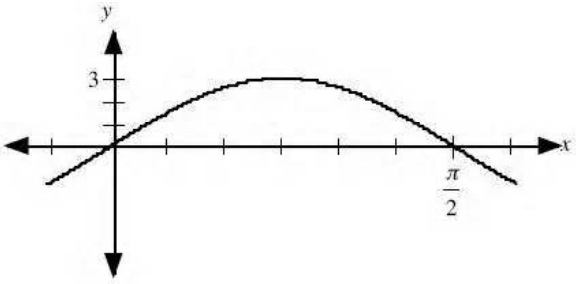
a) $y = a \sin bx$



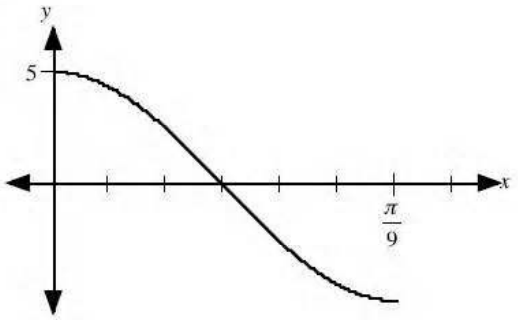
b) $y = a \cos bx$



9. The trigonometric graph shown has a maximum value of 3 and a minimum value of –3.
Determine the equation of the graph in the form $y = a \sin bx$.



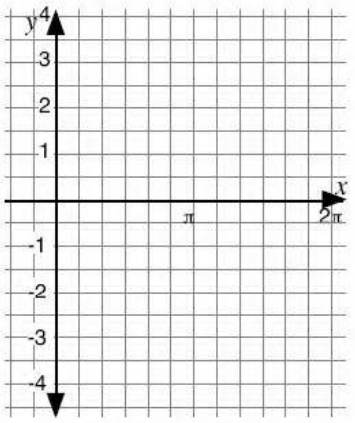
10. The trigonometric graph shown has a maximum value of 5 and a minimum value of –5.
Determine the equation of the graph in the form $y = a \cos bx$.



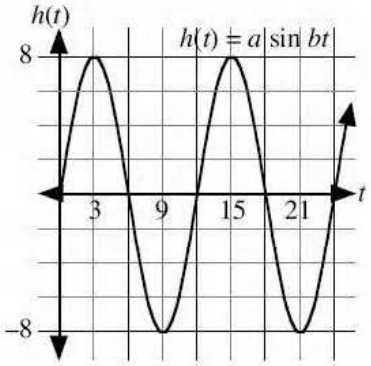
11. Consider the graph of $y = 3 \cos \frac{x}{2}$, $0 \le x \le 2\pi$.

a) State the amplitude and period.

b) Sketch the graph on the grid.
Use a graphing calculator to verify.



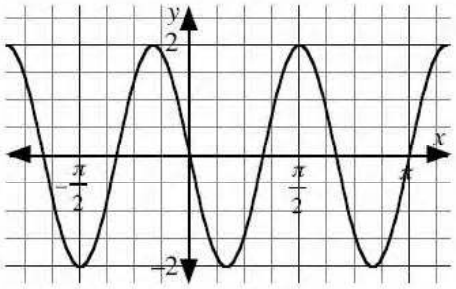
12. The graph represents the change in sea level over a 24 hour period. The graph has equation $h(t) = a \sin bt$, where t is in hours and h is the height, in metres, relative to mean sea level.



a) Determine the equation of the graph.

b) Calculate the height above mean sea level, to the nearest tenth, when $t = 4$.

13. a) Which transformations applied to the graph of $y = \sin x$ would result in the graph shown?



b) Write the equation of the graph in the form $y = a \sin bx$.

Multiple Choice

14. Which of the following functions does not have a period of π?

A. $y = \sin 2x$ **B.** $y = \cos 2x$ **C.** $y = \tan 2x$ **D.** $y = \tan x$

15. Which of the following statements is incorrect?

A. The maximum value of the graph of $y = 3 \cos 2x$ is 3.

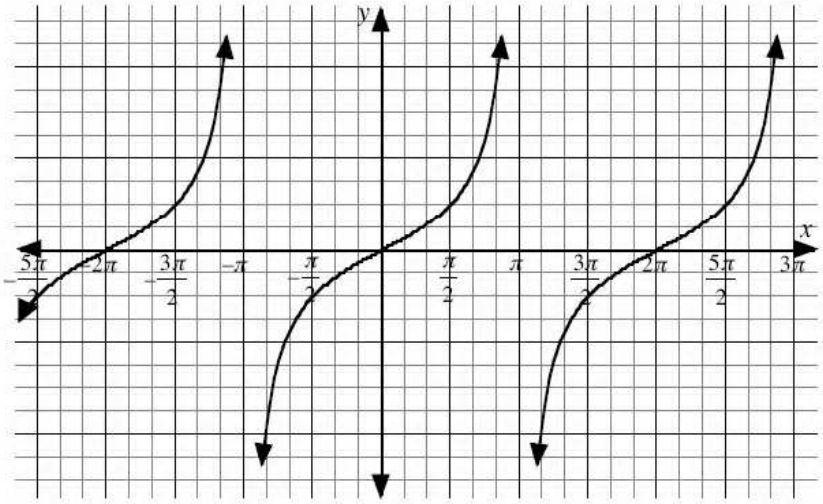
B. The graph of $y = 3 \sin 2x$ has a y-intercept of 3.

C. The graph of $y = 4 \cos 3x$ has an x-intercept of $\frac{\pi}{6}$.

D. The graph of $y = 2 \tan 2x$ has an asymptote with equation $x = \frac{\pi}{4}$.

Numerical Response

16. The graph shown has equation $y = \tan bx$.



The value of b, to the nearest tenth, is _____ .

Answer Key

1. **a)** a vertical expansion by a factor of 5 about the x-axis.

b) a vertical expansion by a factor of 2 about the x-axis and a horizontal expansion by a factor of 2 about the y-axis.

c) a vertical compression by a factor of $\frac{1}{3}$ about the x-axis, a horizontal compression by a factor of $\frac{1}{4}$ about the y-axis, and a reflection in the x-axis.

d) a vertical compression by a factor of 0.2 about the x-axis, a horizontal compression by a factor of $\frac{1}{6}$ about the y-axis, and a reflection in the y-axis.

2. **a)** 5 **b)** 1 **c)** $\frac{7}{3}$ **d)** 4 **3.** **a)** 360° **b)** 60° **c)** 2520° **d)** 270°

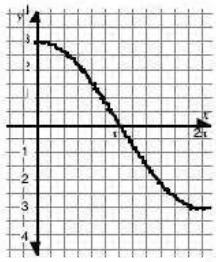
4. **a)** π **b)** $\frac{2\pi}{3}$ **c)** 6π **d)** 2π

5. **a)** $y = 2 \sin \frac{1}{3}x$ **b)** $y = 8 \sin 8x$ **c)** $y = \frac{3}{2} \sin \frac{1}{3}x$

6. **a)** $y = \cos 2x$ **b)** $y = 5 \cos\frac{3}{2}x$ **c)** $y = \frac{5}{3} \cos \frac{2}{3}x$ **7.** **a)** $y = \tan 4x$ **b)** $y = \tan \frac{3}{4}x$

8. **a)** $y = 15 \sin \frac{1}{3}x$ **b)** $y = 4 \cos \frac{1}{4}x$ **9.** $y = 3 \sin 2x$ **10.** $y = 5 \cos 9x$

11. **a)** amp = 3, period = 4π **b)** → → → →



12. **a)** $y = 8 \sin \frac{\pi}{6}t$ **b)** 6.9 metres

13. **a)** a vertical expansion by a factor of 2 about the x-axis, a horizontal compression by a factor of $\frac{1}{3}$ about the y-axis, a reflection in the x- or y-axis.

b) $y = -2 \sin 3x$ or $y = 2 \sin (-3x)$

14. C **15.** B **16.** 0.5