Trigonometry -
Equations, Identities, and Modelling Lesson #10:
Modelling

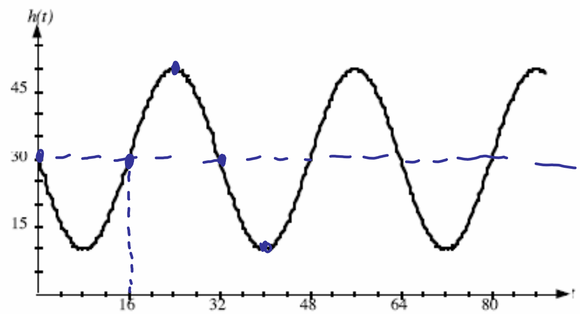
Warm-Up #1 ***Introduction***

If the previous lesson we were asked to solve problems when we were given the equation of a sinusoidal function.

In this lesson we will derive the equation of the sinusoidal function from a graph.

Warm-Up #2 ***Review***

The sinusoidal wave shown has a maximum value of 50 and a minimum value of 10. Write the equation of the sinusoidal wave in the form $h(t) = a \sin[b(t - c)] + d$, where $a > 0$.



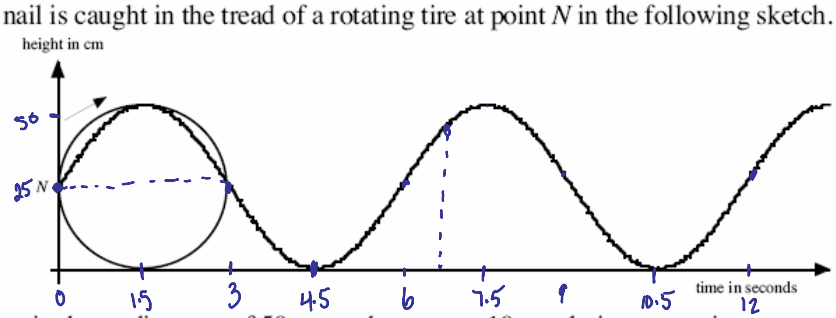
amp $= \frac{50-10}{2} = \frac{40}{2} = 20$

period $= 32$ vertical displacement $d = 30$ hor phase shift $c = 16$

$b = \frac{2\pi}{p} = \frac{2\pi}{32} = \frac{\pi}{16}$

$y = 20 \sin\left[\frac{\pi}{16}(x - 16)\right] + 30$

A nail is caught in the tread of a rotating tire at point N in the following sketch.



$\frac{60}{10} = 6\text{sec}/\text{rev}$

The tire has a diameter of 50 cm and rotates at 10 revolutions per minute. After 4.5 seconds the nail touches the ground.

a) Use the information given to write a scale for each axis.

b) Determine the equation for the height of the nail as a function of time in the form $h(t) = a \sin bt + d$, where $a > 0$.

$a = 25$ $b = \frac{2\pi}{p} = \frac{2\pi}{6} = \frac{\pi}{3}$

$h(t) = 25 \sin \frac{\pi}{3}(t) + 25$

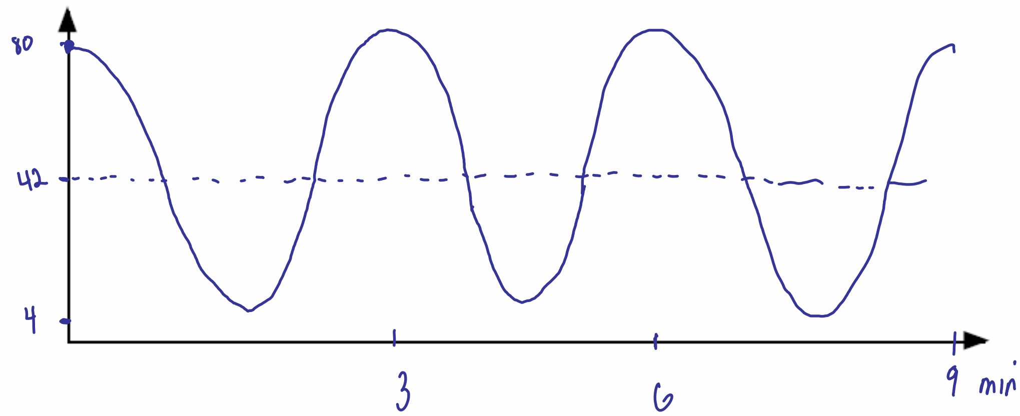
c) How far, to the nearest tenth of a centimetre, is the nail above the ground after 6.5 seconds?

$h(t) = 25 \sin\left[\frac{\pi}{3}(6.5)\right] + 25 = 37.5$ cm

The first ferris wheel ever built was created by a bridge builder by the name of George W. Ferris. in 1893. The diameter of the wheel was approximately 76 metres and the maximum height of the Ferris Wheel was approximately 80 metres. It had 36 carts on the wheel, with each cart able to hold approximately 60 people.

a) If the wheel rotates every three minutes, draw a graph which represents the height of a cart, in metres, as a function of time in minutes. Assume that the cart is at its highest position at $t = 0$. Show three complete cycles.

$76 \div 2$
38



b) Determine the equation of the graph in the form $h(t) = a \cos bt + d$.

$h(t) = 38\cos\left[\frac{2\pi}{3}t\right] + 42$

$a = 38$

$b = \frac{2\pi}{P} = \frac{2\pi}{3}$

$d = 42$

c) How high is the cart 5 minutes after the wheel starts rotating? Answer to the nearest metre.

$h(t) = 38\cos\left[\frac{2\pi}{3}(5)\right] + 42 = 23$ meters

d) How many seconds after the wheel starts rotating does the cart first reach 10 metres from the ground? Answer to the nearest second.

$y_1 = 38\cos\frac{2\pi}{3}t + 42$ $\quad y_2 = 10$ $\quad$ Intersection 1.228 min

$60 + (.228 \times 60)$

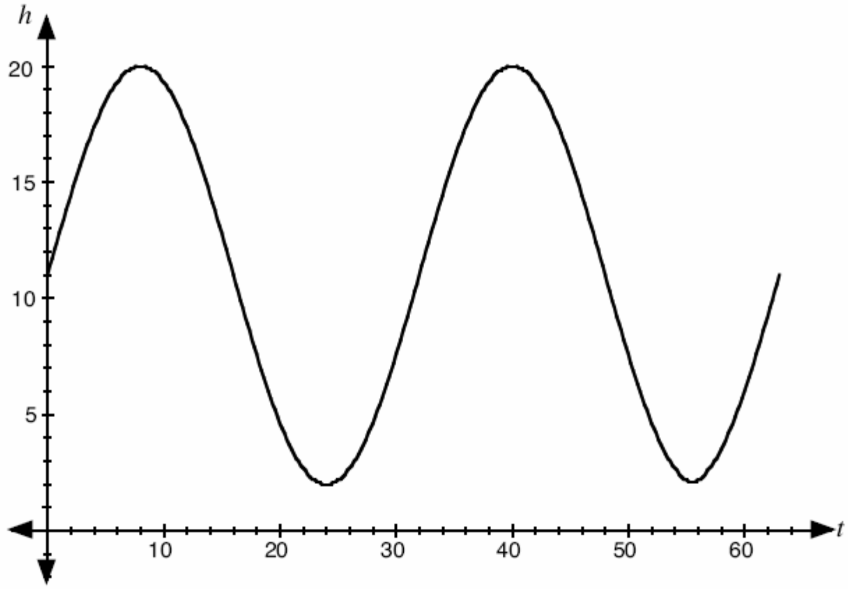
$60 + 14$

74 seconds

Complete Assignment Questions #1 - #6

Assignment

1. The graph shows the height, h metres, above the ground over time, t, in seconds that it takes a person in a chair on a ferris wheel to complete two revolutions. The minimum height of the ferris wheel is 2 metres and the maximum height is 20 metres.



a) How far above the ground is the person as the wheel starts rotating?

b) If it takes 16 seconds for the person to return to the same height, determine the equation of the graph in the form $h(t) = a \sin bt + d$.

c) Find the distance the person is from the ground, to the nearest tenth of a metre, after 30 seconds?

d) How long from the start of the ride does it take for the person to be at a height of 5 metres? Answer to the nearest tenth of a second.

2. A Ferris wheel ride can be represented by a sinusoidal function. A Ferris wheel at Westworld Theme Park has a radius of 15 m and travels at a rate of six revolutions per minute in a clockwise rotation. Ling and Lucy board the ride at the bottom chair from a platform one metre above the ground.

a) Sketch three cycles of a sinusoidal graph to represent the height Ling and Lucy are above the ground, in metres, as a function of time, in seconds.

b) Determine the equation of the graph in the form $h(t) = a \cos [b(t - c)] + d$.

c) If the Ferris wheel does not stop, determine the height Ling and Lucy are above the ground after 28 seconds. Give answer to the nearest tenth of metre.

d) How long after the wheel starts rotating do Ling and Lucy first reach 12 metres from the ground? Give answer to the nearest tenth of a second.

e) How long does it take from the first time Ling and Lucy reach 12 metres until they next reach 12 metres from the ground? Give answer to the nearest second.

3. Andrea, a local gymnast, is doing timed bounces on a trampoline. The trampoline mat is 1 metre above ground level. When she bounces up, her feet reach a height of 3 metres above the mat, and when she bounces down her feet depress the mat by 0.5 metres. Once Andrea is in a rhythm, her coach uses a stopwatch to make the following readings:

- At the highest point the reading is 0.5 seconds.
- At the lowest point the reading is 1.5 seconds.

a) Sketch two periods of the graph of the sinusoidal function which represents Andrea's height above the ground, in metres, as a function of time, in seconds.

b) How high was Andrea above the mat when the coach started timing?

c) Determine the equation of the graph in the form $h(t) = a \sin bt + d$.

d) How high, to the nearest tenth of a metre, was Andrea above the ground after 2.7 seconds?

e) How high, to the nearest tenth of a metre, was Andrea above the mat after 17 seconds?

f) How long after the timing started did Andrea first touch the mat? Answer to the nearest tenth of a second.

4. Consider the following information for a town in Saskatchewan for a leap year of 366 days:

- The latest sunrise time is at 09:00 on December 21 (day 356)
- The earliest sunrise time is at 03:30 on June 21 (day 173)
- There is NO daylight saving time in Saskatchewan
- The sunrise times vary sinusoidally with the day of the year.

a) Write a sinusoidal equation which relates the time of sunrise, t, to the day of the year, d.

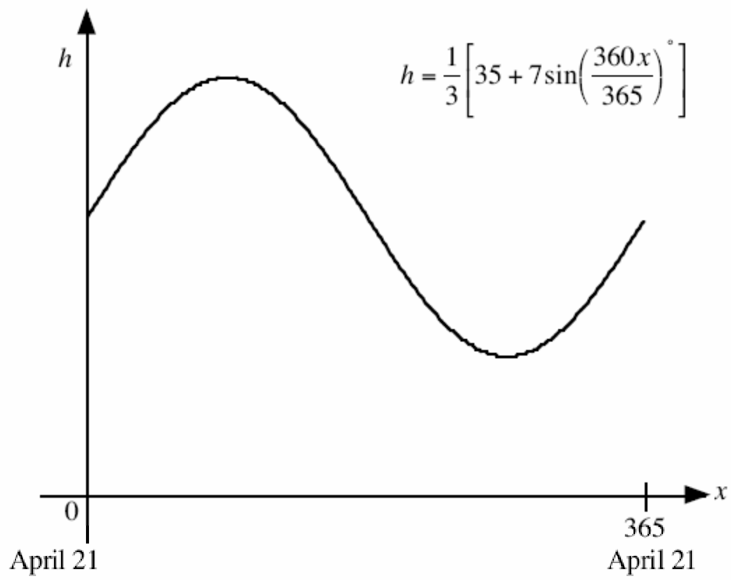
b) Use the equation to determine what time, to the nearest minute, the sun rises on March 11.

c) Determine the average time the sun rises throughout the year.

d) How many days of the year does the sun rise before 6 a.m.?

Use the following information to answer the next two questions

The graph below shows how the number of hours (h) of daylight in a European city changes during the year.



Numerical Response **5.** Mid-winter is the day with the least hours of daylight. The number of hours of daylight, to the nearest tenth of an hour, that there will be on mid-winter's day is _____ .

6. The number of days after April 21 that mid-winter occurs is _____ .

Answer Key

1. **a)** 11 metres **b)** $h(t) = 9\sin\left(\frac{\pi}{16}t\right) + 11$ **c)** 7.6 metres **d)** 19.7 seconds

2. **b)** $h(t) = 15\cos\frac{\pi}{5}(t - 5) + 16$ **c)** 11.4 metres **d)** 2.1 seconds **e)** 6 seconds

3. **b)** 1.25 metres **c)** $h(t) = 1.75\sin\pi t + 2.25$ **d)** 3.7 metres **e)** 1.3 metres **f)** 1.3 seconds

4. **a)** $t = 2.75\cos\left(\frac{\pi}{183}(d + 10)\right) + 6.25$ **b)** 06:45 **c)** 06:15 **d)** 173 **5.** 9.3 **6.** 274