Trigonometry -
Equations, Identities, and Modelling Lesson #1:
Solving First Degree Trigonometric Equations

Warm-Up #1	***Introduction***

In this lesson we will be solving **first degree equations** where the power of the trigonometric function is one (eg. $2 \sin x + 1 = 0$). We will:

- review the algebraic procedure for solving a first degree equation on a domain of length 2π.
- use a **graphical** approach to determine an approximate solution.
- find the **general solution** over the domain of real numbers.

Warm-Up #2	***Review***

Use an algebraic procedure to solve the equation $\sin x = -\frac{1}{2}, \frac{o}{h}$ $0 \le x \le 2\pi$.

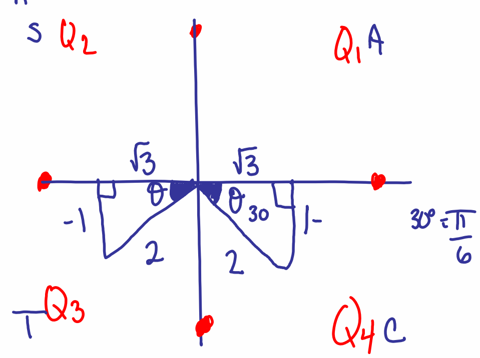
ref $\angle = \frac{\pi}{6}$

Q3 $\theta_1 = \pi + \frac{\pi}{6}$

$\frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$

Q4 $\theta_2 = 2\pi - \frac{\pi}{6}$

$\frac{12\pi}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$



General Solution

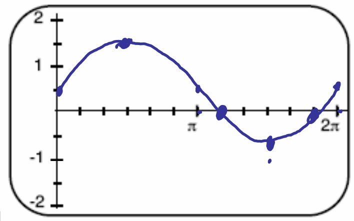
The **general solution** to a trigonometric equation is the solution over the **domain of real numbers**.

Warm-Up #3 — *Exploring a General Solution Using a Graphical Approach*

Consider the equation $\sin x = -\frac{1}{2}$, (i.e. $\sin x + \frac{1}{2} = 0$)

a) Using the following graphical method to estimate the solution to the equation on the domain $0 \le x \le 2\pi$.

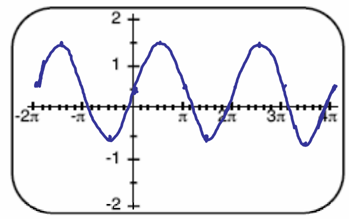
- Use window $x{:}\left|0, 2\pi, \frac{\pi}{6}\right|$ $y{:}[-2, 2, 0.5]$
- Graph $Y_1 = \sin x + \frac{1}{2}$
- Determine (in terms of π), the x-intercepts of graph Y_1 where $0 \le x \le 2\pi$.
- *Solution is* $\frac{7\pi}{6}, \frac{11\pi}{6}$



zeros

b) Using the following graphical method to estimate the solution to the equation on the domain $-2\pi \le x \le 4\pi$.

- Use the window $x{:}\left|-2\pi, 4\pi, \frac{\pi}{6}\right|$ $y{:}[-2, 2, 0.5]$
- Graph $Y_1 = \sin x + \frac{1}{2}$
- Determine the x-intercepts of graph Y_1 where $-2\pi \le x \le 4\pi$. Give the answers as exact values.
- *Solution is* $\frac{-5\pi}{6}, \frac{-\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$



c) Notice that the solution in b) forms two sets of answers which differ by 2π (or a multiple of 2π) from the answers in a). Complete:

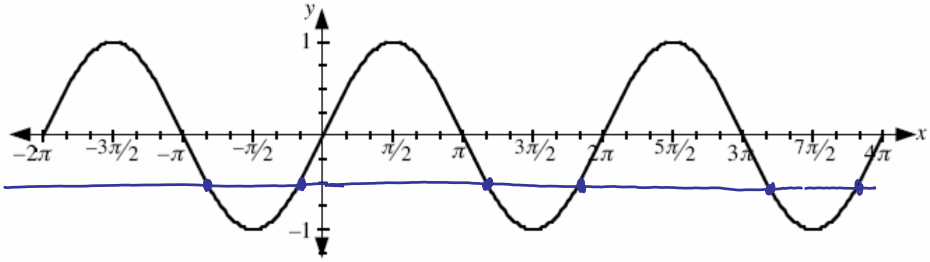
$\frac{-5\pi}{6}, \frac{7\pi}{6}, \frac{19\pi}{6}$ are angles which differ by 2π (or a multiple of 2π)

$\frac{-\pi}{6}, \frac{11\pi}{6}, \frac{23\pi}{6}$ are angles which differ by 2π (or a multiple of 2π)

d) Use this idea to write the general solution to the equation $\sin x = -\frac{1}{2}$ where $x \in \Re$

- *General Solution is* $\frac{7\pi}{6} + 2n\pi, n \in I$ $\frac{11\pi}{6} + 2n\pi, n \in I$

e) The graph of $y_1 = \sin x$ is shown. Show the solution to the equation $\sin x = -\frac{1}{2}$ on the given domain by drawing the line with equation $y_2 = -\frac{1}{2}$ and marking the points of intersection.



pt of intersection

Note The answers in parts b), c) and d) differ by 2π radians because the graph of $y = \sin x$ has a **period** of 2π radians.

Finding a General Solution Using a Graphical Approach

Use the following procedure to find the general solution

1. Use a graphing calculator to solve the equation where the domain is **one period** of the graph of the function (usually $0 \le x \le 2\pi$.). Use either of the following methods:

- enter one side of the equation in $\boxed{Y_1}$ and the other side of the equation in $\boxed{Y_2}$ and find the x-coordinates of the points of intersection.

or

- set the equation equal to zero and enter that into $\boxed{Y_1}$ and find the x-intercepts.

2. Determine the general solution by adding or subtracting **multiples of the period** of the graph of the function.

Class Ex. #1

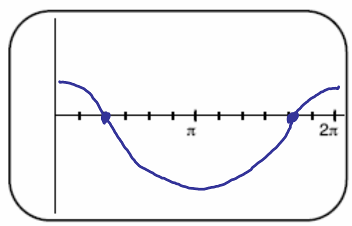
Solve the equation $\cos x - 0.75 = 0,\ x \in \Re$, using two different graphical approaches. Give answers to the nearest hundredth.

Method 1 $y_1 = \cos(x) - .75$

$[0, 2\pi, \frac{\pi}{6}]$

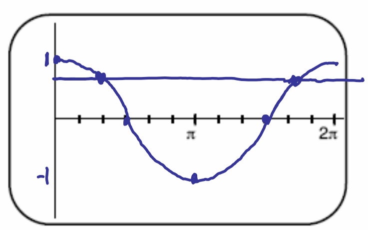
$[-2, 2, .25]$

$x = 0.72$ $x_2 = 5.56$



General Soln $.72 + 2n\pi,\ 5.56 + 2n\pi,\ n \in I$

Method 2 $y_1 = \cos(x)$ $y_2 = .75$



pt of intersection $x = .72$ $x_2 = 5.56$

General Soln $.72 + 2n\pi,\ 5.56 + 2n\pi,\ n \in I$

General Solution Using an Algebraic Approach

Use the following procedure to find the general solution using an algebraic approach:

1. Solve the equation where the domain is **one period** of the graph of the function (usually $0 \le x \le 2\pi$.)

2. The general solution can be determined by adding or subtracting **multiples of the period**.

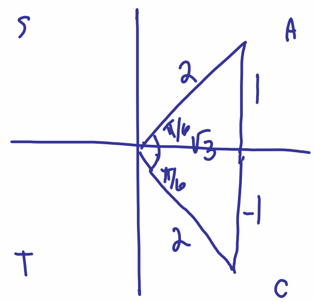
Class Ex. #2

Use an algebraic procedure to find the general solution to the equation $2\cos x - \sqrt{3} = 0, x \in \Re$, where x is in radian measure.

$2\cos x - \sqrt{3} = 0$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$ $\quad \frac{a}{h}$



ref∠ $= \frac{\pi}{6}$ $\quad \theta_1 = 0 + \frac{\pi}{6} = \frac{\pi}{6}$

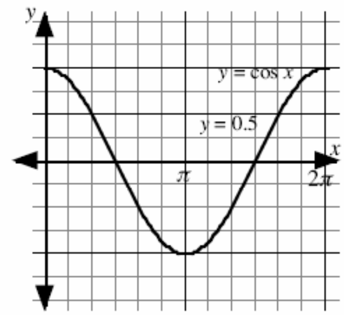
$\theta_2 = 2\pi - \frac{\pi}{6} = \frac{12\pi}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$

general Soln $\quad \frac{\pi}{6} + 2n\pi, \ \frac{11\pi}{6} + 2n\pi, \ n \in I$

Complete Assignment Questions #1 - #13

Assignment

1. The diagram shows the graph of the equations $y = \cos x$ and $y = 0.5$ in $0 \le x \le 2\pi$.

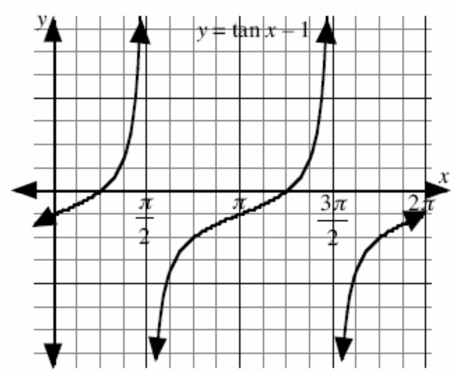


a) **Explain** how to use the graph to determine the approximate solutions to the equation $\cos x = 0.5,\ 0 \le x \le 2\pi$.

b) Write the solutions to the equation $\cos x = 0.5\ ,\ 0 \le x \le 2\pi$. Give solutions as exact values.

c) Write the general solution to the equation $\cos x = 0.5$.

2. The diagram shows the graph of the equation $y = \tan x - 1$ on the domain $0 \le x \le 2\pi$.



a) **Explain** how to use the graph to determine the approximate solutions to the equation $\tan x = 1,\ 0 \le x \le 2\pi$.

b) Write the solutions to the equation $\tan x = 1, 0 \le x \le 2\pi$. Give solutions as exact values.

c) Write the general solution to the equation $\tan x = 1$.

3. Determine the solution to each of the following equations, defined on the domain $0 \le x \le 2\pi$, using a **graphical** approach. Give solutions as exact values.

a) $\sin x = \frac{\sqrt{3}}{2}$ **b)** $\tan x = -1$ **c)** $2 \sec x - 4 = 0$

4. Use the solutions in #3 to write the general solutions to the equations.

a) $\sin x = \frac{\sqrt{3}}{2}$

b) $\tan x = -1$

c) $2 \sec x - 4 = 0$

5. Determine the solution (to the nearest hundredth) to each of the following equations, defined on the domain $0 \le x \le 2\pi$, using a **graphical** approach.

a) $\cos x = 0.6$ **b)** $\cot x = -\frac{1}{2}$ **c)** $\csc x - 3 = 0$

6. Use the solutions in #5 to write the general solutions to the equations.

a) $\cos x = 0.6$

b) $\cot x = -\frac{1}{2}$

c) $\csc x - 3 = 0$

7. Determine the solution to each of the following equations, defined on the domain $0 \le x \le 2\pi$, using an **algebraic** approach.

a) $2 \sin x = -\sqrt{3}$ **b)** $\cot x + \sqrt{3} = 0$ **c)** $3 \sec x - 6 = 0$

8. Use the solutions in #7 to write the general solutions to the equations

a) $2 \sin x = -\sqrt{3}$

b) $\cot x + \sqrt{3} = 0$

c) $3 \sec x - 6 = 0$

9. Use an algebraic approach to determine the general solution to the following equations where x is measured in radians.

a) $2 \cos x - \sqrt{2} = 0$ **b)** $\csc x + 2 = 0$ **c)** $\sqrt{3} \cot x + 1 = 0$

10. Determine the general solution to the following equations where x is in **degree measure**. Answer to the nearest degree.

a) $\cos x = -0.639$ **b)** $\cot x = 0.373$ **c)** $5 \csc x + 6 = 0$

Multiple Choice

11. The general solution to the equation $\csc A + 2 = 0$ is

A. $A = \frac{\pi}{6} + n\pi,\ n \in I$

B. $A = \frac{\pi}{6} + 2n\pi,\ \frac{5\pi}{6} + 2n\pi,\ \ n \in I$

C. $A = \frac{7\pi}{6} + n\pi,\ \frac{11\pi}{6} + n\pi,\ \ n \in I$

D. $A = \frac{7\pi}{6} + 2n\pi,\ \frac{11\pi}{6} + 2n\pi,\ \ n \in I$

12. The general solution to the equation $\sqrt{3}\cot\theta - 1 = 0$ is

A. $\theta = \frac{\pi}{6} + n\pi,\ \ n \in I$

B. $\theta = \frac{\pi}{6} + 2n\pi,\ \frac{7\pi}{6} + 2n\pi,\ \ n \in I$

C. $\theta = \frac{\pi}{3} + n\pi,\ \ n \in I$

D. $\theta = \frac{\pi}{3} + 2n\pi,\ \frac{4\pi}{3} + 2n\pi,\ \ n \in I$

Numerical Response

13. The smallest positive solution to the equation $\sec x - 5 = 0$, correct to the nearest tenth of a radian, is $x =$ _____

Answer Key

1. a) Find the x-coordinates of the points of intersection of the two graphs

b) $x = \frac{\pi}{3}, \frac{5\pi}{3}$ **c)** $x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in I$

2. a) Find the x-intercepts of the graph **b)** $x = \frac{\pi}{4}, \frac{5\pi}{4}$ **c)** $x = \frac{\pi}{4} + n\pi, n \in I$

3. a) $x = \frac{\pi}{3}, \frac{2\pi}{3}$ **b)** $x = \frac{3\pi}{4}, \frac{7\pi}{4}$ **c)** $x = \frac{\pi}{3}, \frac{5\pi}{3}$

4. a) $x = \frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, n \in I$

b) $x = \frac{3\pi}{4} + n\pi, n \in I$

c) $x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in I$

5. a) $x = 0.93, x = 5.36$ **b)** $x = 2.03, x = 5.18$ **c)** $x = 0.34, x = 2.80$

6. a) $x = 0.93 + 2n\pi, 5.36 + 2n\pi, \ n \in I$
b) $x = 2.03 + n\pi, 5.18 + n\pi, \ n \in I$
c) $x = 0.34 + 2n\pi, 2.80 + 2n\pi, \ n \in I$

7. a) $x = \frac{4\pi}{3}, \frac{5\pi}{3}$ **b)** $x = \frac{5\pi}{6}, \frac{11\pi}{6}$ **c)** $x = \frac{\pi}{3}, \frac{5\pi}{3}$

8. a) $x = \frac{4\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in I$

b) $x = \frac{5\pi}{6} + n\pi, n \in I$

c) $x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in I$

9. a) $x = \frac{\pi}{4} + 2n\pi, \frac{7\pi}{4} + 2n\pi, n \in I$

b) $x = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi, n \in I$

c) $x = \frac{2\pi}{3} + n\pi, n \in I$

10. a) $x = 130° + 360n°, \ 230° + 360n° \ \ n \in I$
b) $x = 70° + 180n°, \ n \in I$
c) $x = 236° + 360n°, \ 304° + 360n° \ \ n \in I$

11. D **12.** C **13.** 1.4