Trigonometry -
Equations, Identities, and Modelling Lesson #3:
Equations which require a Graphical Solution

Equations which require a Graphical Solution

Some equations cannot be solved by an algebraic approach. In these cases we use a graphical approach to estimate the solution.

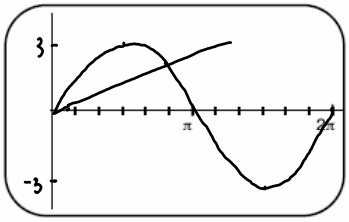
Warm-Up

Consider the equation $3 \sin x = x$.

a) Use the following graphical method to estimate the solution to the equation **on the domain $0 \leq x \leq 2\pi$.**

- Choose a window with appropriate values which will enable us to find the points of intersection of the graphs of $y = 3 \sin x$ and $y = x$.

  Use window x: $\left| 0, 2\pi, \frac{\pi}{6} \right|$ y: [-4, 4, 1]



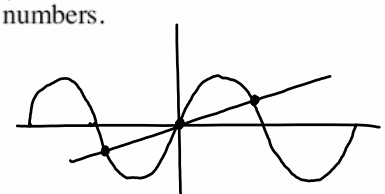
- Graph $Y_1 = x$
- Graph $Y_2 = 3 \sin x$
- Use INTERSECT to determine, to the nearest tenth, the x-coordinate(s) of the point(s) of intersection.

  x= 0.0 , 2.3

b) To determine the **general solution** to the equation $3 \sin x = x$ we need to recognize that the diagonal line ($y = x$) and the periodic function ($y = 3 \sin x$) will intersect once more to the left of the origin and nowhere else on the domain of real numbers.

- Write the general solution to the equation $3 \sin x = x$.

  x= -2.3, 0.0 , 2.3



c) Describe how we could find approximate solutions to the equation $3 \sin x = x$ using the ZERO feature of the calculator.

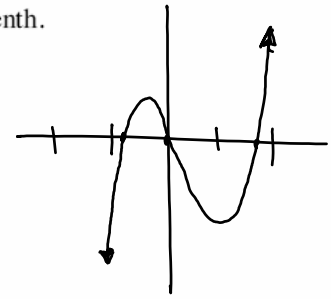
Graph $y = \sin x - x$

Find x-int of the graph using calc func. zero feature.

Use a graphical approach and the ZERO feature of the calculator to find the solution to the equation $x^3 - x^2 = 2\sin x$. Give the answers to the nearest tenth.

$y_1 = x^3 - x^2 - 2\sin x$

$x = -0.9, 0.0, 1.7$



Complete Assignment Questions #1 - #7

Assignment

1. Use a graphical approach to solve the following equations to the nearest hundredth where necessary.

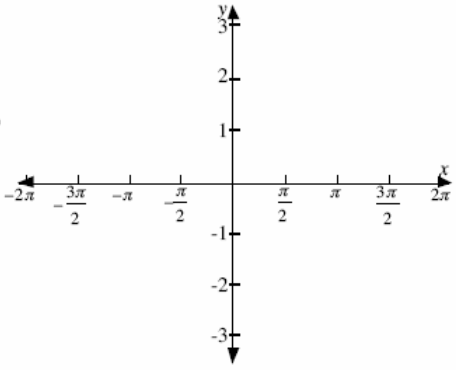
a) $\cos x = x + 1$

b) $x - 2\sin x = 0$

c) $\sin 2x = x$,

d) $\cos x = x^2$

2. **a)** Graph $y = 4\sin x - x$ on a domain $-2\pi \le x \le 2\pi$. Show the graph on the grid.

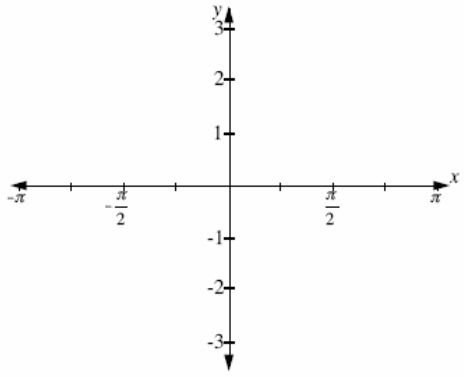


b) Solve the equation $x = 4\sin x$, where $x \in \Re$, to the nearest hundredth.

c) Explain how you could use the graph to solve the equation $x = 4\sin x + 3$, $-2\pi \le x \le 2\pi$.

d) Solve the equation $x = 4\sin x + 3$ where $x \in \Re$, to the nearest hundredth.

3. **a)** Graph $y = x^2 + \sin 6x$ on a domain $-\pi \le x \le \pi$.
Show the graph on the grid.



b) Solve the equation $x^2 = -\sin 6x$ where $x \in \Re$, to the nearest hundredth.

c) Explain how you could use the graph to solve the equation $x^2 = -\sin 6x + 1, -\pi \le x \le \pi$.

d) Solve the equation $x^2 = -\sin 6x + 1$ where $x \in \Re$, to the nearest hundredth.

Multiple Choice

4. How many solutions are there to the equation $\cos^3 x = \sin^3 x + 0.5$ where $-\pi \le x \le \pi$?

A. 1

B. 2

C. 3

D. more than 3

5. How many solutions are there to the equation $\cos 3x + \frac{1}{2}x = 0$?

A. 2

B. 3

C. 4

D. more than 4

Numerical Response

6. The smallest positive solution to the equation $\sec x - x = 0$, correct to the nearest tenth of a radian, is $x =$ _____ .

7. The number of solutions to the equation $x \sin 2x = 0$ where $-2\pi \le x \le 2\pi$ is _____ .

Answer Key

1. **a)** 0 **b)** –1.90, 0, 1.90 **c)** –0.95, 0, 0.95 **d)** –0.82, 0.82

2. **b)** –2.47, 0.00, 2.47
c) Find the x coordinates of the points of intersection of the graph with the line $y = -3$
d) 3.11

3. **b)** –0.48, 0.00, 0.58, 0.89
c) Find the x coordinates of the points of intersection of the graph with the line $y = 1$
d) –1.38, –1.07, –0.63, 0.21, 0.34, 1.04

4. B **5.** B **6.** 4.9 **7.** 9