Trigonometry - Equations, Identities, and Modelling Lesson #8: Double Angle Identities

Warm-Up #1

Consider the statement $\sin 2\theta = 2 \sin \theta$.

a) Determine whether or not the statement can be verified using $\theta = \frac{\pi}{6}$.

$LS = \sin 2\left(\frac{\pi}{6}\right) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad RS = 2\sin \frac{\pi}{6} = 2\left(\frac{1}{2}\right) = 1$ $\quad LS \neq RS$ not verified

b) What can we say about the statement $\sin 2\theta = 2 \sin \theta$?

not true

Warm-Up #2

Use exact values to verify the following statements:

a) $\sin \frac{\pi}{3} = 2 \sin \frac{\pi}{6} \cos \frac{\pi}{6}$

$\frac{\sqrt{3}}{2} = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ $\quad LS = RS$

b) $\cos \frac{\pi}{2} = \cos^2 \frac{\pi}{4} - \sin^2 \frac{\pi}{4}$

$0 = \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$0 = \frac{1}{2} - \frac{1}{2}$

$0 = 0$ $\quad LS = RS.$

Double Angle Identities

Warm-Up #2 is an example of verifying the double angle identities. The double angle identities for sine and cosine are shown.

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

These identities are on the formula sheet.

The identity for $\cos 2\theta$ can also be written in the following forms, which are given on the formula sheet. You will be asked to prove all three forms of the identity in the assignment.

$$\cos 2\theta = 2 \cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2 \sin^2\theta$$

These identities are on the formula sheet.

Class Ex. #1

Use an addition identity to prove the double angle identity $\sin 2A = 2 \sin A \cos A$.

$\sin(A+A) = \sin A \cos A + \cos A \sin A$

$= 2 \sin A \cos A$ $\qquad$ LS = RS

Class Ex. #2

Consider the identity $\dfrac{2 \tan x}{1 + \tan^2 x} = \sin 2x$.

a) Describe how to use a graphing calculator to verify the identity.

graph $y_1 = \dfrac{2\tan x}{1+\tan^2 x}$ $\quad$ $y_2 = \sin 2x$ $\quad$ graphs are identical

b) Prove the identity.

$$\text{LS. } \frac{\frac{2\sin x}{\cos x}}{\frac{\cos^2 x}{\cos^2 x} 1 + \frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{2\sin x}{\cos x}}{\frac{\cancel{\cos^2 x + \sin^2 x}^{\,1}}{\cos^2 x}} = \frac{2\sin x}{\cancel{\cos x}} \times \frac{\cos^{\cancel{2}} x}{1} = 2\sin x \cos x = \sin 2x$$

Class Ex. #3

Express each of the following in terms of a single trigonometric function.

a) $2 \sin 4x \cos 4x$

$= \sin 2(4x) = \sin 8x$

b) $\cos^2\left(\frac{1}{2}A\right) - \sin^2\left(\frac{1}{2}A\right)$

$= \cos 2\left(\frac{1}{2}A\right) = \cos A$

c) $\sin \frac{5}{2}x \cos \frac{5}{2}x$ $\qquad \frac{1}{2}\left(2 \sin\left(\frac{5}{2}x\right)\cos\left(\frac{5}{2}x\right)\right)$

$= \frac{1}{2} \sin 2\left(\frac{5}{2}x\right) = \frac{1}{2}\sin 5x$

Solving Equations Using Double Angle Identities

Class Ex. #4

Jenny graphed the following equations on her graphing calculator.

$y = -\cos 2x$

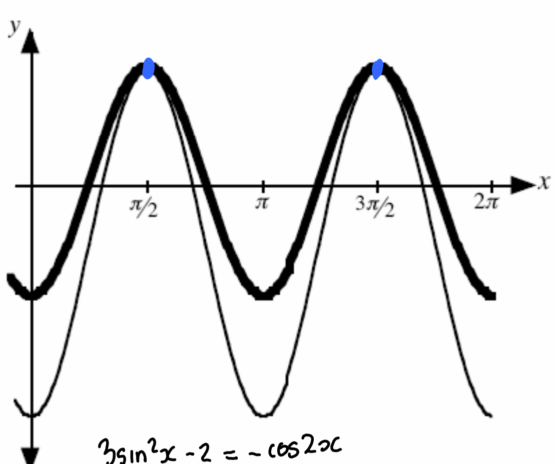
$y = 3 \sin^2 x - 2$

a) Describe how to use the graphs to solve the equation

$3 \sin^2 x + \cos 2x - 2 = 0$

where $0 \le x \le 2\pi$.

Mark these points with **DOTS** on the grid.



$3\sin^2 x - 2 = -\cos 2x$

Determine points of intersection of the two curves

b) Find the solution to the equation $3\sin^2 x + \cos 2x - 2 = 0$, where $0 \le x \le 2\pi$, by algebraic means, as exact values.

$3\sin^2 x + (1 - 2\sin^2 x) - 2$

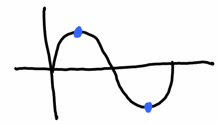
$\sin^2 x - 1 = 0$

$\sqrt{\sin^2 x} = \sqrt{1}$

$\sin x = \pm 1$

$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$



c) Describe how to use the graphs to solve the equation
$-\cos 2x\,(3\sin^2 x - 2) = 0$, where $0 \le x \le 2\pi$.
Mark these points with a **SQUARE** on the grid.

$\cos 2x = 0$ or $3\sin^2 x - 2 = 0$ Determine the x-intercepts of each curve.

d) Solve algebraically the equation $-\cos 2x\,(3\sin^2 x - 2) = 0$, where $0 \le x \le 2\pi$, to the nearest tenth.

$-\cos 2x = 0$

$\cos 2x = 0$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi + \frac{\pi}{2}, 2\pi + \frac{3\pi}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$x = 0.8, 2.4, 3.9, 5.5$

$3\sin^2 x = 2$ $\sin^2 x = \frac{2}{3}$ $\sin x = \pm\sqrt{\frac{2}{3}}$ ref $\angle = 0.955$

$x = .955, \pi - .955, \pi + .955, 2\pi - .955$

$x = 1.0, 2.2, 4.1, 5.3$

$x = 0.8, 1.0, 2.2, 2.4, 3.9, 4.1, 5.3, 5.5$

Complete Assignment Questions #1 - #8

Assignment

1. Prove the following double angle identities using tan addition identity for cosine.

a) $\cos 2\theta = \cos^2\theta - \sin^2\theta$. **b)** $\cos 2\theta = 2\cos^2\theta - 1$ **c)** $\cos 2\theta = 1 - 2\sin^2\theta$.

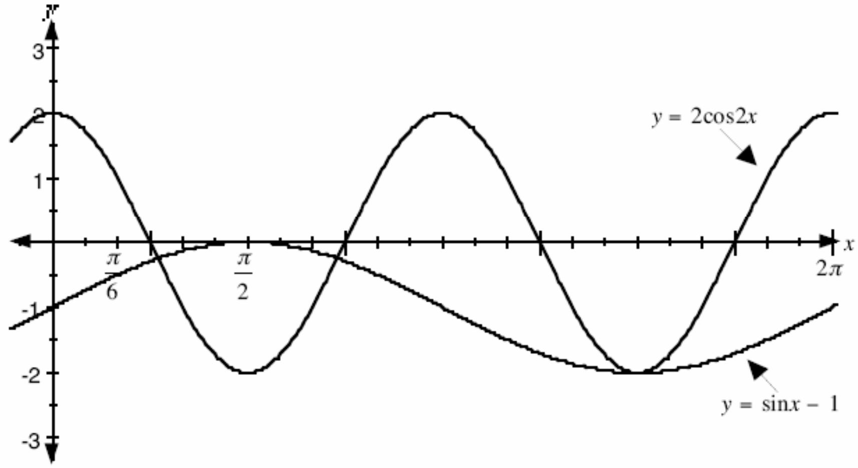
2. Prove the identity $\dfrac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta$.

3. Solve the following equations for $0 \le x \le 2\pi$.

a) $\cos 2x + \cos x = 0$ **b)** $\cos 2x = 1 - 2 \sin x$ **c)** $2 \cos^2 \frac{1}{2}x - 1 = 0$

d) $\sin 2x + \cos x = 0$ **e)** $\cos 2x - \sin x = 0$

4. The diagram shows the graphs of two trig functions
$y = 2\cos 2x$ and $y = \sin x - 1$ for $0 \le x \le 2\pi$.



a) Describe how to use the graphs to solve the equation
$2\cos 2x - \sin x + 1 = 0$, where $0 \le x \le 2\pi$.
Mark these points with **DOTS** on the grid.

b) Find the solution to the equation $2\cos 2x - \sin x + 1 = 0$, where $0 \le x \le 2\pi$, by algebraic means, to the nearest hundredth.

c) Describe how to use the graphs to solve the equation
$2\cos 2x(\sin x - 1) = 0$, where $0 \le x \le 2\pi$.
Mark these points with a **SQUARE** on the grid.

d) Solve algebraically the equation $2\cos 2x(\sin x - 1) = 0$, where $0 \le x \le 2\pi$.
Give the answers as exact values.

5. Express each of the following in terms of a single trigonometric function.

a) $2\sin\frac{1}{2}x\cos\frac{1}{2}x$ **b)** $\cos^2 2A - \sin^2 2A$ **c)** $1 - 2\sin^2 3x$

6. Use a double angle identity to simplify and evaluate

a) $2\sin 15° \cos 15°$ **b)** $\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12}$ **c)** $\sin\frac{5\pi}{12}\cos\frac{5\pi}{12}$

Multiple Choice

7. The expression $\dfrac{\cos^2\frac{3}{2}x - \sin^2\frac{3}{2}x}{\sin\frac{3}{2}x\ \cos\frac{3}{2}x}$ is equivalent to

A. $\cos\frac{3}{2}x - \sin\frac{3}{2}x$

B. $\cot 3x$

C. $2\cot 3x$

D. $2\csc 3x$

Numerical Response

8. If $a\cos^2\frac{\pi}{8} - a\sin^2\frac{\pi}{8} = 4\sqrt{2}$, the value of a, to the nearest tenth, is _____ .

Answer Key

3. a) $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ **b)** $x = 0, \frac{\pi}{2}, \pi, 2\pi$ **c)** $x = \frac{\pi}{2}, \frac{3\pi}{2}$ **d)** $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

e) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

4. a) Find the x-coordinates of the points of intersection of the two graphs

b) 0.85, 2.29, 4.71 **c)** Find the x-intercepts of each graph **d)** $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

5. a) $\sin x$ **b)** $\cos 4A$ **c)** $\cos 6x$

6. a) $\sin 30° = \frac{1}{2}$ **b)** $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ **c)** $\frac{1}{2}\sin\frac{5\pi}{6} = \frac{1}{4}$ **7.** C **8.** 8.0