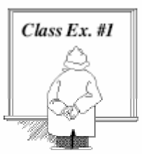
Trigonometry - Equations, Identities, and Modelling Lesson #9: Sinusoidal Functions

Sinusoidal Functions

A function whose graph resembles the sine or cosine curve is called a sinusoidal function. The graph of a sinusoidal function is called a sinusoidal graph. Many periodic phenomena have sinusoidal graphs, eg. the time of sunrise as a function of the day of the year, the height of a chair of a ferris wheel as a function of time, the depth of the ocean due to changing tides as a function of time, etc.

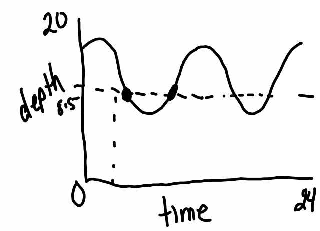
In this lesson the equation of the sinusoidal function will be given. In the next lesson we will derive the equation of the sinusoidal function from given information.

Most of the equations used will be functions of time and the variable used will be t. The period of the graph will be in time units. Graphical methods will be used to solve problems and determining a suitable window is an essential feature of the solution.



The minimum depth, d metres, of water in a harbour, t hours after midnight, can be approximated by the function $d(t) = 12 + 5\cos 0.5t$, where $0 \le t \le 24$.

$d(t) = 5\cos 0.5t + 12$ VD=12 AMP=5



a) Determine the maximum and minimum depths of water in the harbour.

max = 12+5 = 17m

min = 12-5 = 7m

b) Determine the period of the function.

$p = \frac{2\pi}{b} = \frac{2\pi}{.5} = 4\pi$ hours

c) Write a suitable window which can be used to display the graph of the function.

[0, 24, 2] [0, 20, 2]

d) What is the depth of water, to the nearest tenth of a metre at 2:00 a.m.?

Value x=2 y= 14.7m

e) A ship which requires a minimum of 8.5 metres of water is in harbour at midnight. By what time, to the nearest minute, must it leave to prevent grounding?

$y_2 = 8.5$ X= 4.6923 hours 4:41am

.6923 x 60 = 41m

f) What is the next time, to the nearest necessary minute, that the ship can return to the harbour?

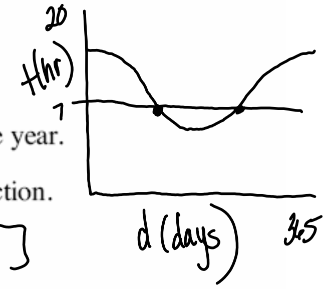
$y_2 = 8.5$ $x_2 = 7.8739$ 7:53am

.8736 x 60 = 53 min

Class Ex. #2

In a certain town in British Columbia, the time of sunrise for any day can be found using the formula

$$t = -1.79 \sin\left(2\pi\frac{(d-78)}{365}\right) + 6.3$$



where t is the time in hours after midnight and d is the number of the day in the year.

a) Write a suitable window which can be used to display the graph of the function.

max $= 6.3 + 1.79 = 8.09$ $[0, 365, 30]$ $[0, 10, 2]$
min $= 6.3 - 1.79 = 4.51$

b) Use the formula to determine, to the nearest minute, when the sun rose on May 7, the 127th day of the year.

Value $x = 127$ $y = 4.96292$ $.96292 \times 60 = 58$ min 4:5

c) Determine on which days of the year the sun rose at 7 a.m.

$y_2 = 7$ $x_1 = 54.659 = 55^{th}$ day $x_2 = 283.84029 = 284^{th}$ day

Complete Assignment Questions #1 - #5

Assignment

1. The alarm in a noisy factory is a siren whose volume, V decibels fluctuates so that t seconds after starting, the volume is given by the function $V(t) = 18 \sin \frac{\pi}{15} t + 60$.

a) What are the maximum and minimum volumes of the siren?

b) Determine the period of the function.

c) Write a suitable window which can be used to display the graph of the function.

d) After how many seconds, to the nearest tenth, does the volume first reach 70 decibels?

e) The background noise level in the factory is 45 decibels. Between which times, to the nearest tenth of a second, in the first cycle is the alarm siren at a lower level than the background noise?

f) For what percentage, to the nearest per cent, of each cycle is the alarm siren audible over the background factory noise?

2. A top secret satellite is launched into orbit from a remote island not on the equator. When the satellite reaches orbit, it follows a sinusoidal pattern that takes it north and south of the equator, (i.e. the equator is used as the horizontal axis). Twelve minutes after it is launched it reaches the farthest point north of the equator. The distance north or south of the equator can be represented by the function

$$d(t) = 5000\cos\left[\frac{\pi}{35}(t-12)\right]$$

where $d(t)$ is the distance of the satellite north of the equator t minutes after being launched.

a) How far north or south of the equator is the launch site? Answer to the nearest km.

b) Is the satellite north or south of the equator after 20 minutes? What is this distance to the nearest kilometre?

c) When, to the nearest tenth of a minute, will the satellite first be 2500 km south of the equator?

3. The height of a tidal wave approaching the face of the cliff on an island is represented by the equation

$$h(t) = 7.5\cos\left(\frac{2\pi}{9.5}t\right)$$

where $h(t)$ is the height, in metres, of the wave above normal sea level t minutes after the wave strikes the cliff.

a) What are the maximum and minimum heights of the wave relative to normal sea level?

b) What is the period of the function?

c) How high, to the nearest tenth of a metre, will the wave be, relative to normal sea level, one minute after striking the cliff?

d) Normal sea level is 6 metres at the base of the cliff.

i) For what values of h would the sea bed be exposed?

ii) How long, to the nearest tenth of a minute, after the wave strikes the cliff does it take for the sea bed to be exposed?

iii) For how long, to the nearest tenth of a minute, is the sea bed exposed?

4. The depth of water in a harbour can be represented by the function

$$d(t) = -5\cos\frac{\pi}{6}t + 16.4$$

where $d(t)$ is the depth in metres and t is the time in hours after low tide.

a) What is the period of the tide?

b) A large cruise ship needs at least 14 metres of water to dock safely. For how many hours per cycle, to the nearest tenth of an hour, can a cruise ship dock safely?

5. A city water authority determined that, under normal conditions, the approximate amount of water, $W(t)$, in millions of litres, stored in a reservoir t months after May 1, 2003, is given by the formula $W(t) = 1.25 - \sin\frac{\pi}{6}t$.

a) Sketch the graph of this function over the next three years.

b) The authority decided to carry out the following simulation to determine if they had enough water to cope with a serious fire.

"If, on November 1, 2004, there is a serious fire which uses 300 000 litres of water to bring under control, will the reservoir run dry if water rationing is not imposed?"

i) Explain how to use the graph in a) to solve the problem.

ii) Will the reservoir run dry if water rationing is not imposed? If so, in what month will this occur?

Answer Key

1. a) max = 78 dB, min = 42 dB, **b)** 30 s **c)** x: [0, 40, 5] y: [30, 100, 10] answers may vary
d) 2.8 s **e)** 19.7 s - 25.3 s **f)** 81%

2. a) 2369 km north **b)** 3765 km north **c)** 35.3 minutes

3. a) max = 7.5 m, min = –7.5 m **b)** 9.5 min **c)** 5.9 m
d) i) $h \leq -6$ **ii)** 3.8 min **iii)** 1.9 min

4. a) 12 h **b)** 7.9 h

5. b) i) After $t = 18$ move the graph down 0.3 units. If the graph falls below the t axis, the reservoir will run dry. ***or*** Draw the line $y = 0.3$. If the line intersects the graph, the reservoir will run dry.
ii) In month 26, i.e. July, 2005