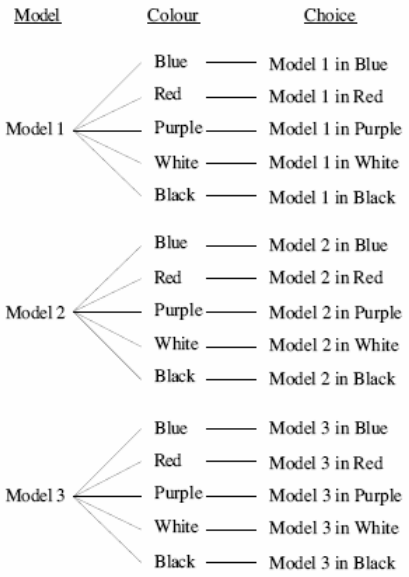
Permutations and Combinations Lesson #1:
The Fundamental Counting Principle

Warm-Up

Blanche is shopping for a car. The dealer tells her that he has 3 different models and that each model is available in 5 different colours.

a) Use the tree diagram to determine how may different choices Blanche has.

b) Look at the number of choices there are for the model and the number of choices there are for the colour. How do you use these numbers to arrive at the answer in a)?



The Fundamental Counting Principle

Consider a task made up of several stages. If the number of choices for the first stage is a, the number of choices for the second stage is b, the number of choices for the third stage is c, etc., then the number of ways in which the task can be completed is $a \times b \times c \times \ldots$.
This is called the **fundamental counting principle**.

A toy manufacturer makes a wooden toy in three parts:

Part 1: the top part may be coloured red, white or blue, 3
Part 2: the middle part may be orange or black, 2
Part 3: the bottom part may be yellow, green, pink or purple. 4

Determine how many different coloured toys can be produced using:

a) a tree diagram

Top Middle Bottom

Red — Orange — yellow, green, pink, purple
Red — Black — yellow, green, pink, purple
White — Orange — yellow, green, pink, purple
White — Black — yellow, green, pink, purple
Blue — Orange — yellow, green, pink, purple
Blue — Black — yellow, green, pink, purple

b) the fundamental counting principle

$3 \times 2 \times 4 = 24$ different coloured toys
top middle bottom

Determine the number of distinguishable four letter arrangements that can be formed from the word **ENGLISH** if;

7 letters

a) letters can be repeated? $7 \cdot 7 \cdot 7 \cdot 7 = 7^4 = 2401$

b) no letters are repeated and:

i) there are no further restrictions? $7 \cdot 6 \cdot 5 \cdot 4 = 840$

ii) the first letter must be **E**? $\underset{E}{1} \cdot 6 \cdot 5 \cdot 4 = 120$

iii) the "word" must contain **G**?

G _ _ _ $(6 \cdot 5 \cdot 4)(4)$

_ G _ _ $120(4) = 480$

_ _ G _

_ _ _ G

iv) the first and last letters must be vowels? $\underset{V}{2} \cdot 5 \cdot 4 \cdot \underset{V}{1} = 40$

A Math 30 quiz consists of eight multiple choice questions. Each question has four choices A, B, C or D. How many different sets of answers are possible?

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^8 = 65536$

The telephone numbers allocated to subscribers in a rural area consist of one of the following:

- the digits 345 followed by any three further digits, *or*,
- the digit 2 followed by one of the digits 1 to 5 followed by any three further digits.

How many different telephone numbers are possible?

$\underset{345}{1} \cdot 10 \cdot 10 \cdot 10$ or $\underset{2}{1} \cdot \underset{1\text{-}5}{5} \cdot 10 \cdot 10 \cdot 10$

$1000 + 5000 = 6000$ different numbers

Car number plates in an African country consist of a letter other than I or O followed by three digits, the first of which cannot be zero, followed by any two letters which are not repeated. How many different car number plates can be produced?

$\underset{\text{letter not I or O}}{24} \cdot 9 \cdot 10 \cdot 10 \cdot \underset{\text{letter with no rep.}}{26 \cdot 25} = 14,040,000$ different car number plates.

Consider the digits 2, 3, 5, 6, 7 and 9. 6 options

a) If repetitions are not permitted, how many 3-digit numbers can be formed? $6 \cdot 5 \cdot 4 = 120$

b) How many of these are

i) less than 400? $2 \cdot 5 \cdot 4 = 40$ ($\neq 5,6,7,9$) < 400

ii) even? $5 \cdot 4 \cdot 2 = 40$ (even)

iii) odd? $5 \cdot 4 \cdot 4 = 80$ (odd)

iv) multiples of 5? $5 \cdot 4 \cdot 1 = 20$ (multiple of 5)

Consider a six-digit numeral.(Note: 022713 is classified as the 5-digit numeral 22713).

a) How many odd six-digit numerals have no repeating digits?

$8 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 5 = 67200$

$\neq 0$; odd

b) How many even six-digit numerals have no repeating digits?

$9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 1$ or $8 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$ (2, 4, or 8)

15120 ; zero $+ \neq 0$; 53760

$= 68,880$

Complete Assignment Questions #1 - #14

Assignment

1. A football team has the following kit :

jersey: red or black
pants: white, red or black
socks: red or white

If the team plays in a different uniform each week, for how many weeks can it play before it has to repeat a previous uniform?

2. The score at the end of the second period of a hockey game is: Canucks 6 Oilers 3.
How many different possibilities are there for the score at the end of the first period?

3. With the new renovations completed at Prestwick High School, there will be seven entrances. In how many different ways can a student coming for Math tutorials;

a) enter the school and exit through a different entrance?

b) enter and exit through any entrance?

c) enter and exit through the same entrance?

4. How many ways are there of getting from A to C in each diagram, passing through each point at most once?

Answer to Diagram 1

Answer to Diagram 2

Answer to Diagram 3

Diagram 1

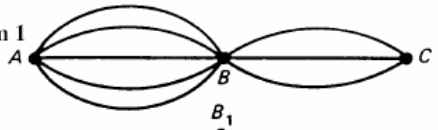


Diagram 2

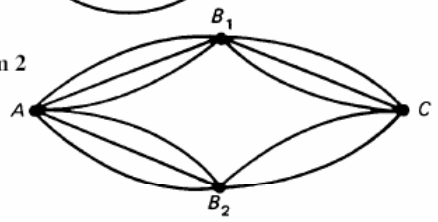
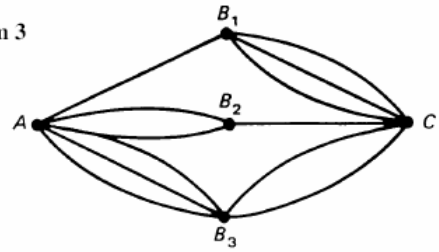


Diagram 3



5. Find the number of four letter "words" that can be formed from the letters of the word **PRODUCE** if;

a) each letter can only be used once:

b) each letter can only be used once and the "word" must:

i) contain only consonants?

ii) begin and end with a consonant?

iii) begin with a vowel?

iv) contain the letter **P**?

v) begin with **D** and end with a vowel?

6. In a class of 30 students, how many ways are there of awarding;

a) a first prize, a second prize and a third prize in Mathematics?

b) a Mathematics prize, a Chemistry prize and a Physics prize?
(Assume that each of the students is capable of winning any of the prizes.)

7. How many even four digit numerals have no repeated digits?

8. A vehicle license plate consists of 3 letters followed by 3 digits. How many different license plates are possible if:

a) there are no restrictions on the letters or digits used?

b) no letters may be repeated?

c) the first digit cannot be zero and no digits can be repeated?

9. **a)** How many different three-digit numerals can be formed from the digits 1, 5 and 8 if the digits cannot be repeated?

b) How many different three-digit numerals can be formed using the digits 1, 3, 5, 7 and 9 if the digits may be repeated?

c) How many four-digit numerals can be formed from the digits 0, 2 and 3 if the digits may be repeated? (Note: 0223 is classified as the 3-digit numeral 223).

d) How many different non-zero numerals are possible using some or all of the numerals 0, 1, 2, and 3 if the digits cannot be repeated?

10. How many different sums of money can be made from two pennies, four nickels, two quarters, and five dollar coins?

11. Mr. and Mrs. McDonald want a family picture taken with their children, Hamish, Flora and James. In how many different ways can all five line up in a straight line for the picture if;

a) there are no restrictions?

b) the parents must be at either end of the line?

c) baby James must be in the middle?

d) the children alternate with the adults?

12. Determine the number of distinguishable four letter arrangements that can be formed from the word **PRODUCT** given the following restrictions;

a) no restrictions

b) the "word" begins with **PR**?

c) the "word" has two vowels in the middle?

d) the "word" has two consonants in the middle?

13. Ocean going ships have use coloured flags hung vertically for signalling. By changing the order of the coloured flags, the ships can send out different signals.

If ships carry six different coloured flags, one flag of each colour, how many different signals are possible if;

a) all six flags are used?

b) four flags are used?

c) at least two flags are used?

Numerical Response

14. Sandra is taking an examination which consists of two parts, A and B, with the following instructions.

- Part A consists of three questions and the student must do two.
- Part B consists of four questions and the student must do two.
- Part A must be completed before starting Part B.
- At the end of the exam the student has to list the order in which she attempted the questions.

The number of different possible orders is _____ .

Answer Key

1. 12 **2.** 28 **3. a)** 42 **b)** 49 **c)** 7

4. Diagram 1 → 15 **Diagram 2** → 15 **Diagram 3** → 11

5. a) 840 **b)** **i)** 24 **ii)** 240 **iii)** 360 **iv)** 480 **v)** 60

6. a) 24360 **b)** 27000 **7.** 2296

8. a) 17 576 000 **b)** 15 600 000 **c)** 11 389 248

9. a) 6 **b)** 125 **c)** 54 **d)** 48 **10.** 269

11. a) 120 **b)** 12 **c)** 24 **d)** 12 **12. a)** 840 **b)** 20 **c)** 40 **d)** 400

13. a) 720 **b)** 360 **c)** 1950 **14.** 72