# Permutations and Combinations Lesson #4:
Combinations Part 1

Warm-Up #1

As part of the grade 12 English course, students are required to read the following three books in a three month period:

"The Grapes of Wrath", "The Wars", "The Bean Trees".

a) Due to previous late returns Steve is only allowed to sign out one English book from the school library per month.
   List all the different orders in which Steve could sign out the books.
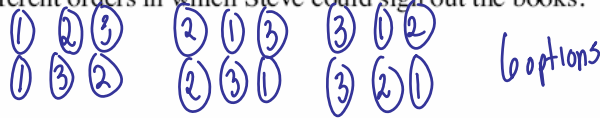
b) Tariq is allowed to sign out all three books at the same time.
   How many different ways can he sign out all three books at the same time?

Part a) is an example of a permutation where the order is important.
Part b) is an example of a combination where the order is NOT important.

Combinations

A **selection** of a set of objects in which the **order of the selection is NOT important** is called a **combination**.

Warm-Up #2

Suppose that the students in Warm-Up #1 were required to read only two of the books.

a) Complete the table to show the number of ways in which Steve and Tariq could do this:

Steve (Permutations)	Tariq (Combinations)
The Grapes of Wrath, The Wars	The Grapes of Wrath, The Wars
The Grapes of Wrath, The Bean Trees	
The Wars, The Grapes of Wrath	

b) Complete the following statement:

• The number of combinations is equal to the number of permutations divided by ___2___ or ___2___ factorial.

Warm-Up #3

Five students, Al, Byron, Colin, Dave and Eric take part in a cross country race to represent their school.

a) Suppose the winner of the race wins $50, the runner-up wins $25 and third place is $10.

The table below shows all possible ways in which the three prizes could be awarded to the five participants in the race.

"A" stands for Al, "B" for Byron, "C" for Colin, "D" for Dave, "E" for Eric.

ABC	ABD	ABE	ACD	ACE	ADE	BCD	BCE	BDE	CDE
ACB	ADB	AEB	ADC	AEC	AED	BDC	BEC	BED	CED
BAC	BAD	BAE	CAD	CAE	DAE	CBD	CBE	DBE	DCE
BCA	BDA	BEA	CDA	CEA	DEA	CDB	CEB	DEB	DEC
CAB	DAB	EAB	DAC	EAC	EAD	DBC	EBC	EBD	ECD
CBA	DBA	EBA	DCA	ECA	EDA	DCB	ECB	EDB	EDC

- Is this an example of permutations or combinations?   *Permutation*

- How many ways are there to award the three prizes? $_5P_3 = 60$

b) For participating in the cross-country race, the school has been awarded three places at a running clinic. The school coach decides to select the 3 lucky students from the ones who took part in the cross country race.

- Use the table from a) (which has been duplicated below) to circle the different ways the three students can be chosen.

ABC	ABD	ABE	ACD	ACE	ADE	BCD	BCE	BDE	CDE
ACB	ADB	AEB	ADC	AEC	AED	BDC	BEC	BED	CED
BAC	BAD	BAE	CAD	CAE	DAE	CBD	CBE	DBE	DCE
BCA	BDA	BEA	CDA	CEA	DEA	CDB	CEB	DEB	DEC
CAB	DAB	EAB	DAC	EAC	EAD	DBC	EBC	EBD	ECD
CBA	DBA	EBA	DCA	ECA	EDA	DCB	ECB	EDB	EDC

- Is this an example of permutations or combinations?   *Combinations*

- How many ways are there to select the three students? $\dfrac{_5P_3}{3!} = 10$

c) Complete the following statement:

- The number of combinations is equal to the number of permutations divided by _6_ or _3_ factorial.

Combinations of "n" different objects taken "r" at a time   (r ≤ n)

The Warm-Ups are examples of the following general rule:

$$_nC_r = \frac{_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

The number of combinations of "n" items taken "r" at a time is

$$_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \quad *$$

This formula is on the formula sheet

Note • The $_nC_r$ key on a calculator can be used to evaluate combinations.

Class Ex. #1 Three students from a class of ten are to be chosen to go on a school trip.

a) In how many ways can they be selected?
   Write the answer in factorial notation and evaluate.

$$_{10}C_3 = \frac{10!}{3!(10-3)!} = \frac{10!}{3! \cdot 7!} = \frac{10 \times 9 \times 8 \times 7!}{3 \times 2 \times 1 \times 7!} = \frac{720}{6} = 120$$

b) Confirm the answer in a) using the $_nC_r$ key on a calculator. $_{10}nCr_3 = 120$

Class Ex. #2 To win the LOTTO 649 a person must correctly choose six numbers from 1 to 49. Jasper, wanting to play the LOTTO 649, began to wonder how many numbers he could make up. How many choices would Jasper have to make to ensure he had the six winning numbers?

$$_{49}nCr_6 = 13\ 983,816$$

Class Ex. #3 The Athletic Council decides to form a sub-committee of seven council members to look at how funds raised should be spent on sports activities in the school. There are a total of 15 athletic council members, 9 males and 6 females. The sub-committee must consist of exactly 3 females.

a) In how many ways can
   the females be chosen? $_6C_3 = 20$

b) In how many ways can
   the males be chosen? $_9C_4 = 126$

c) In how many ways can
   the sub-committee be chosen?

$$_6C_3 \times _9C_4 = 20 \times 126 = 2520$$

d) In how many ways can the sub-committee
   be chosen if Bruce, the football coach must
   be included?

$$\frac{1}{B} \times _6C_3 \times _8C_3 = 1(20)(56) = 1120$$

Class Ex. #4

A standard deck of 52 cards has the following characteristics:
- 4 suits (spades, clubs, diamonds, and hearts).
- Each suit has 13 cards.
- Two suits are black (spades and clubs).
- Two suits are red (diamonds and hearts).
- Face cards are considered to be Jacks, Queens, and Kings.

Poker is a card game played from a deck of 52 cards.

a) How many different five card poker hands are possible?

$$_{52}C_5 = 2,598,960$$

b) In how many of the hands in a) will there be:

i) all diamonds?

$$_{13}C_5 = 1287$$

ii) 4 black cards and 1 red card?

$$_{26}C_4 \times {}_{26}C_1$$
$$388\,700$$

iii) 3 kings and 2 queens?

$$_4C_3 \times {}_4C_2$$
$$24$$

iv) 3 kings?

$$_4C_3 \times {}_{48}C_2$$
$$4512$$

v) four aces?

$$_4C_4 \times {}_{48}C_1$$
$$48$$

vi) 5 cards of the same suit? (called a "flush")

$$4 \times {}_{13}C_5$$
$$4(1287) = 5148$$

Complete Assignment Questions #1 - #12

Assignment

1. Pete's Perfect Pizza Company has 9 choices of topping available.

    a) How many different 2-topping pizzas can be made?

    b) How many different 3-topping pizzas can be made?

2. A theatre company consisting of 6 players is to be chosen from 15 actors. How many selections are possible if the company must include Mrs. Jones?

3. How many different rectangles can be formed by eight horizontal lines and three vertical lines?

4. Edinburgh High School has a twelve-member student council. A four member sub-committee is to be selected to organize dances.

    a) How many different sub-committees are possible?

    b) How many four member sub-committees are possible if the council president and vice-president must be members?

5. A basketball coach has five guards and seven forwards on his basketball team.

    a) In how many different ways can he select a starting team of two guards and three forwards?

    b) How many different starting teams are there if the star player, who plays guard, must be included?

6. Twelve face cards are removed from a deck of fifty-two cards. From the face cards, three card hands are dealt. Determine the number of distinct three card hands that are possible which include:

    a) no restrictions   **b)** 3 kings

    c) 1 Queen and 2 kings   **d)** only 1 Jack

7. Consider a standard deck of 52 cards. Determine the number of distinct six card hands that are possible which include:

    a) no restrictions   **b)** only clubs   **c)** 2 clubs and 4 diamonds

    d) no sevens   **e)** 4 tens   **f)** only 1 Jack and 4 Queens

8. Explain the meaning of $_{10}C_2$. Why does $_2C_{10}$ not make sense?

9. Develop a problem where $\begin{pmatrix} 9 \\ 4 \end{pmatrix}$ would be applicable as a solution.

10. The number of ways that an executive committee consisting of prime minister, deputy prime minister, treasurer, and secretary can be chosen from 16 student council members is

    A.   4!

    B. $\dfrac{16!}{4!}$

    C. $\dfrac{16!}{12!\,4!}$

    D. $\dfrac{16!}{12!}$

11. There are three girls and six boys on the school softball team. The team consists of a pitcher, a catcher, four infielders, and three outfielders. How many ways can the nine different positions be filled if the pitcher must be a girl and the catcher must be a boy?

    A. $_3C_1 \times {_6C_1} \times 7!$
    B. $_3C_1 \times {_6C_1} \times 9!$
    C. $_3C_1 \times {_6C_1}$
    D. $\dfrac{9!}{3!\,6!}$

12. Sarah is one of a group of eight people from which a committee of four people must be formed. The number of different committees possible if Sarah must sit on the committee is _____ .

Answer Key

1. a) 36   b) 84   2. 2002   3. 84   4. a) 495   b) 45
5. a) 350   b) 140   6. a) 220   b) 4   c) 24   d) 112
7. a) 20 358 520   b) 1716   c) 55 770   d) 12 271 512   e) 1128   f) 176
8. The number of ways of selecting 2 items from 10 where the order of selection is not important. You cannot select 10 items from 2.
9. Answers may vary   10. D   11. A   12. 35