Permutations and Combinations Lesson #5:
Combinations Part 2

Combinations Problems with "at least", "at most", etc.

The Student Council decides to form a sub-committee of five council members to look at how funds raised should be spent on the students of the school. There are a total of 11 student council members, 5 males and 6 females.
How many different ways can the sub-committee consist of;

a) exactly three females?

(F)(M)

$_6C_3 \cdot {}_5C_2 = 200$ ways

b) at least three females?

3F 2M or 4F 1M or 5F No M

$_6C_3\ {}_5C_2 + {}_6C_4\ {}_5C_1$ or $_6C_5\ {}_5C_0$

200 + 75 + 6 = 281 ways

c) at least one female? 1F 4M or 2F 3M or 3F 2M or 4F 1M or 5F 1M OR

* Compliment Total − No female

$_{11}C_5$ − No F 5 Male $_6C_0\ {}_5C_5$

462 − 1 = 461 ways

Consider a standard deck of 52 cards. How many different five card hands can be formed containing:

a) at least 1 red card?

1R 4B or 2R 3B or 3R 2B or 4R 1B or 5R No B

(Total) − OR (No Red)(5 Black)

$_{52}C_5 - {}_{26}C_0 \cdot {}_{26}C_5 = 2\,533\,180$ ways

b) at most 2 kings?

No Kings or 1 King or 2 Kings

$_{40}C_5\ {}_{48}C_5 + {}_4C_1\ {}_{48}C_4 + {}_4C_2\ {}_{48}C_3$

2 594 400 ways

c) exactly two pairs?

1st Pair 2nd Pair 5th Card what # for the pair

$_4C_2 \cdot {}_4C_2 \cdot {}_{44}C_1 \cdot {}_{13}C_2 = 123\,552$

Complete Assignment Questions #1 - #6

A class consists of 5 girls and 7 boys. A committee is to be formed consisting of 2 girls and 3 boys. In how many ways can a teacher choose the committee if:

a) there are no further restrictions?

$_5C_2 \cdot {}_7C_3 = 350$ ways
Girls Boys

b) Johnny, the Principal's son, has to be on the committee?

$_1C_1 \; {}_5C_2 \; {}_6C_2 = 150$ ways
Johnny Girls Boy

c) the twins, Peter and Paul, cannot both be on the committee?

No restrictions 350 ways.
Peter & Paul were on the Committee
$_1C_1 \cdot {}_1C_1 \cdot {}_5C_2 \cdot {}_4C_1 = 50$
Peter Paul Girls Boys

$350 - 50 = 300$

Combinations which are Equivalent

Jane calculated $_{10}C_2$ to be 45 arrangements. She then calculated $_{10}C_8$ to be 45 arrangements.

a) Explain in words why $_{10}C_2 = {}_{10}C_8$.

If you choose 2 there are 8 leftover
If you choose 8 there are 2 leftover

⇓

b) Use factorial notation to show that $_{10}C_2 = {}_{10}C_8$.

$\frac{10!}{(10-2)!\,2!} = \frac{10!}{(10-8)!\,8!}$ $\qquad \frac{10!}{8!\,2!} = \frac{10!}{2!\,8!}$

c) Give another two examples of equivalent combinations.

$_{12}C_3 = {}_{12}C_9$ $\qquad 220 = 220$

$_8C_3 = {}_8C_5$ $\qquad 56 = 56$

d) Prove the identity $_nC_r = {}_nC_{n-r}$.

$\frac{n!}{(n-r)!\,r!} \;\Big|\; \frac{n!}{[n-(n-r)]!\,(n-r)!}$

$\frac{n!}{(n-n+r)!\,(n-r)!} = \frac{n!}{r!\,(n-r)!}$

LS = RS

Solving for "n" in Combination Problems

During a Pee Wee hockey tryout, all the players met on the ice after the last practice and shook hands with each other. How many players attended the tryouts if there were 300 handshakes in all?

$\frac{n!}{(n-r)!\,r!}$

$n C_2 = 300$ $\quad \frac{n!}{(n-2)!\,2!} = 300$ $\quad \frac{(n)(n-1)(n-2)!}{(n-2)!\cdot 2} = 300(2)$

$n^2 - n = 600$

$n^2 - n - 600 = 0$

$(n-25)(n+24)$

$n = 25 \quad n = -24$

Accept

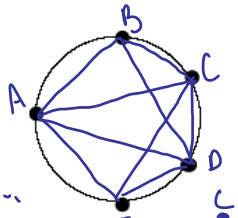
Polygons and Diagonals

Warm-Up

Consider circles with five points marked on the circumference.

a) How many triangles can be formed using these five points? Write the answer in combination notation.

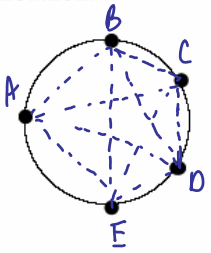
$5C3$



ABC, ABD, ABE, ...

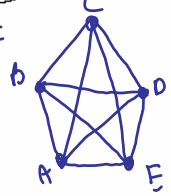
b) How many lines can be drawn connecting two points on the circle? Write the answer in combination notation.

$5C2$



AB, AC, AD, AE
BC, BD, BE
CD, CE
DE

c) Sketch a regular pentagon.



i) How many lines can be drawn connecting two points on the pentagon, including the sides of the pentagon?

10

ii) How many of these lines are diagonals of the pentagon?

5

iii) Express the answer to ii) in terms of combinations.

$5C2 - 5$

d) How many diagonals are there in a regular octagon?

$8C2 - 8 = 20$

e) How many diagonals are there in a regular n sided polygon?

$nC2 - n$

The number of diagonals in a regular n-sided polygon is

$$ {}_nC_2 - n $$

*This formula is **NOT** on the formula sheet*

A polygon has 65 diagonals. How many sides does it have?

$${}_nC_2 - n = 65 \qquad \frac{n!}{(n-2)!2!} - n = 65 + n \qquad \frac{(2)n(n-1)(n-2)!}{(n-2)!\,2} = (65+n)2$$

$$n^2 - n - 2n = 130 + 2n \qquad n^2 - 3n - 130 = 0 \qquad (n-13)(n+10) = 0$$

$n = 13$ Accept $\quad n = -10$ reject

The polygon has 13 sides

Complete Assignment Questions #7 - #16

Assignment

1. The Athletic Council decides to form a sub-committee of 6 council members to look at a new sports program. There are a total of 15 student council members, 6 females and 9 males. How many different ways can the sub committee consist of at most one male?

2. A group of 4 journalists is to be chosen to cover a murder trial. There are 5 male and 7 female journalists available. How many possible groups can be formed:
   a) consisting of 2 men and 2 women? **b)** consisting of at least one woman?

3. Consider a standard deck of 52 cards. How many different four card hands have;
   a) at least one black card? **b)** at least 2 kings?

   c) two pairs **d)** at most 2 clubs?

4. City Council decides to form a sub-committee of five aldermen to investigate transportation concerns. There are 4 males and 7 females. How many different ways can the sub- committee be formed consisting of at least one female member?

5. A basketball squad of 11 players is to be chosen from 17 available players. In how many ways can this be done if:

a) Colin and Darryl must be selected? **b)** Jeff and Brent cannot both be selected?

6. An all-night showing at a movie theatre is to consist of five movies. There are fourteen different movies available, ten disaster movies and four horror movies. How many possible schedules include:

a) at least one horror movie? **b)** at least four disaster movies?

c) both "Airport Disaster" and "Halloween Horror"?

7. The number of ways that a selection of 7 students can be chosen from a class of 28 is the same as the number of ways that n students can be chosen from the same class. What is the value of n ?

8. How many people are there in a class in which there are 20 ways to select a committee of three people?

9. Solve for n.

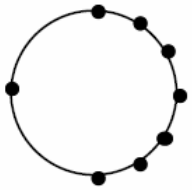
a) ${}_nC_3 = 84$ **b)** ${}_{11}C_n = 330$ (two answers)

c) ${}_nC_7 = {}_{n+1}C_8$

10. How many diagonals are there in each?

a) a hexagon **b)** a decagon **c)** an p-sided polygon

11. There are eight visible points on the circle below. How many triangles can be made using these eight points?



12. If 35 quadrilaterals can be placed on a circle with a series of points on it, then how many points are on the circle?

Multiple Choice

13. After everyone had shaken hands once with everyone else in a room, there was a total of 66 handshakes. How many people were in the room?

A. 11
B. 12
C. 33
D. 67

Numerical Response

14. A basketball team consists of some guards and six forwards. If there are 420 ways to select two guards and three forwards to the starting line-up, then the number of guards on the team is _____ .

15. Collinear points are points which share the same straight line. The number of triangles which can be formed from 10 points if no three of the points are collinear is _____ .

16. There are 170 diagonals in a polygon. The number of sides of the polygon is _____ .

Answer Key

1. 55 **2.** **a)** 210 **b)** 490
3. **a)** 255 775 **b)** 6961 **c)** 2808 **d)** 258 856
4. 462 **5.** **a)** 5005 **b)** 7371
6. **a)** 1750 **b)** 1092 **c)** 220 **7.** 21 **8.** 6
9. **a)** 9 **b)** 4 or 7 **c)** 7
10. **a)** 9 **b)** 35 **c)** $\frac{p(p-3)}{2}$ **11.** 56 **12.** 7 **13.** B
14. 7 **15.** 120 **16.** 20