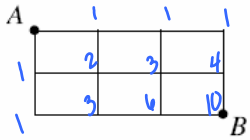
Permutations and Combinations Lesson #7:
Pathway Problems

Warm-Up

Consider the following problem:

"Find the number of pathways from A to B if paths must always move closer to B"



This problem can be solved in several ways.

a) Solve this problem by tracing the number of paths on the grid.

b) Explain how this problem can be regarded as an example of permutations with repetitions. Determine the number of pathways using this approach.

A to B travel 5 block lengths, A to B comprised of 3 blocks right and 2 blocks down

Same as the number of permutations of a 5 letter word 3R and 2D $\frac{5!}{3!2!} = 10$ pathways

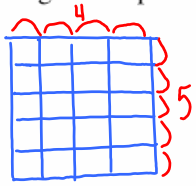
c) Explain how this problem can be regarded as an example of combinations. Use the combination formula or Pascal's Triangle to determine the number of pathways

see sketch

A city centre has a rectangular road system with 5 streets running north to south and 6 avenues running west to east.

a) Draw a grid to represent this situation.

N



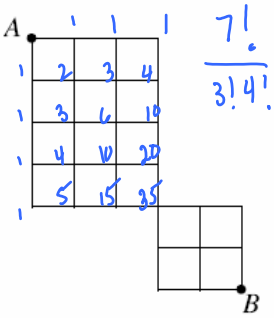
$4+5=9$ $\therefore \frac{9!}{4!5!}$

b) Sean is driving a car and is situated at the extreme northwest corner of the city centre. In how many ways can he drive to the extreme southeast corner if at each turn he moves closer to his destination (assume all streets and avenues allow two-way traffic).

$\frac{9!}{4!5!} = 126$ ways

Find the number of pathways from A to B if paths must always move closer to B.

a) A

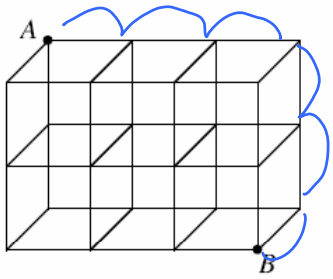


$\frac{7!}{3!4!} = 35$

$\frac{4!}{2!2!} = 6$

$35 \times 6 = 210$ pathways

b) A

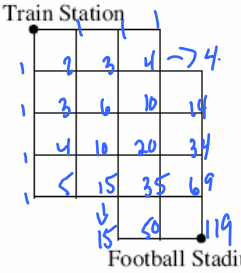


$\frac{6!}{3!2!}$

$= 60$ pathways

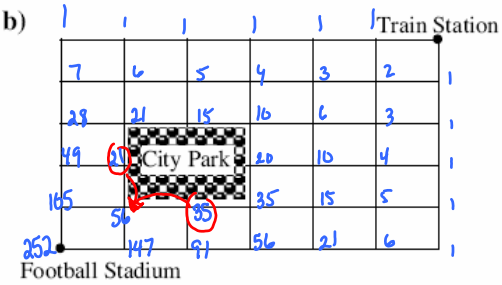
A taxi company is trying to find the quickest route during rush hour traffic from the train station to the football stadium. How many different routes must be considered if at each intersection the taxi must always move closer to the football stadium?

a) Train Station



119 different ways

b) Train Station

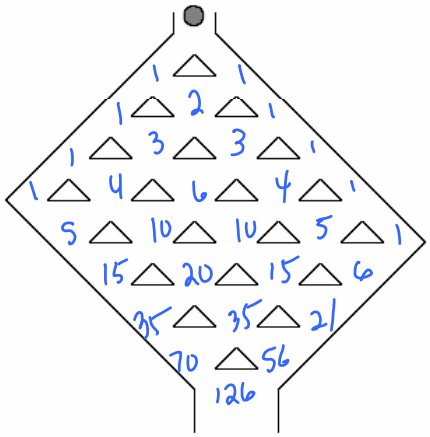


252 different ways

How many different paths can a pinball take as it falls from top to bottom?

$\frac{9!}{5!4!}$

$= 126$ paths

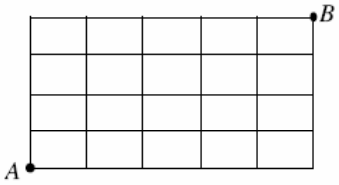


Complete Assignment Questions #1 - #7

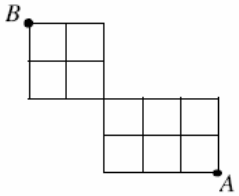
Assignment

1. Find the number of pathways from A to B if paths must always move closer to B.

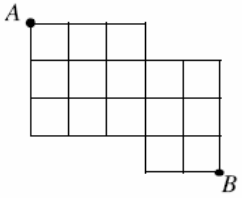
a)



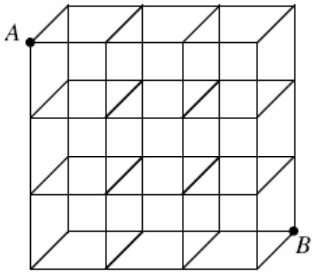
b)



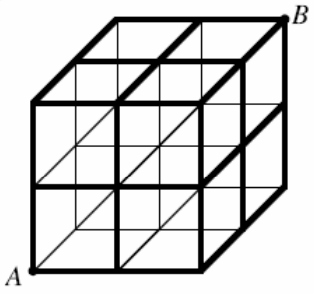
c)



d)



e)



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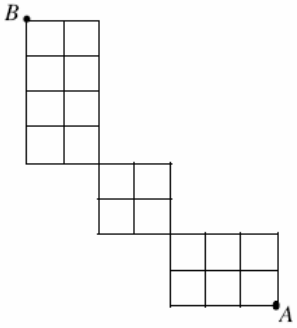
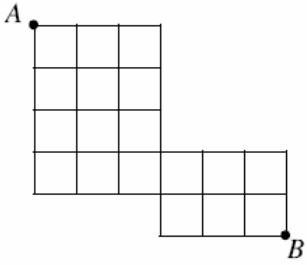
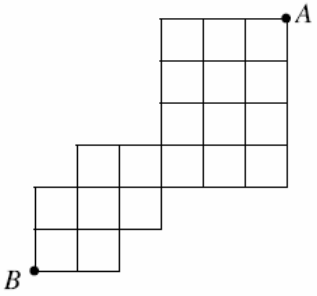


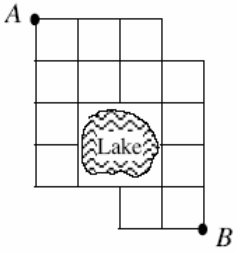
g)



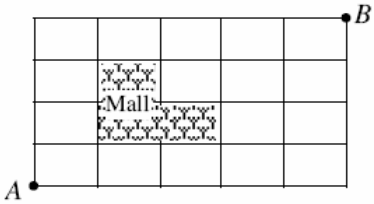
h)



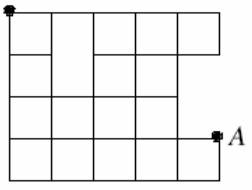
i)



j)



k)

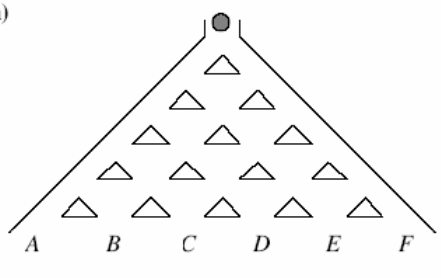


2. The bakery is four blocks south and three blocks west of the supermarket. The bakery driver, bored with travelling the same route, decides to use a different route for each delivery. Assuming that he always travels closer to the supermarket, how many deliveries are possible before he has to repeat a route?

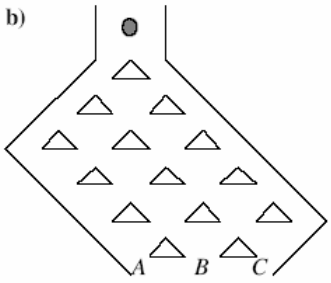
3. A town has 10 streets running from north to south and 8 avenues running from west to east. A man wishes to drive from the extreme south-west intersection to the extreme north-east intersection, moving either north or east along one of the streets or avenues. Find the number of routes he can take. State any assumptions that you have made.

4. Given the following pinball machines determine the number of pathways to reach each of the exits.

a)



b)

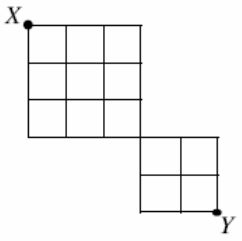


Multiple Choice

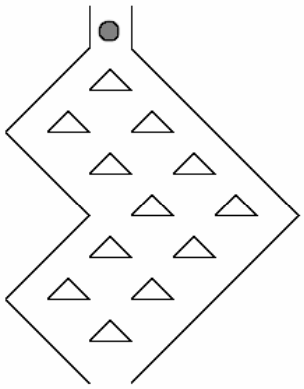
5. The number of pathways from X to Y if paths must always move closer to Y is

A. $\frac{6!}{3!\,3!} + \frac{4!}{2!\,2!}$ **B.** $\frac{6!}{3!\,3!} \times \frac{4!}{2!\,2!}$

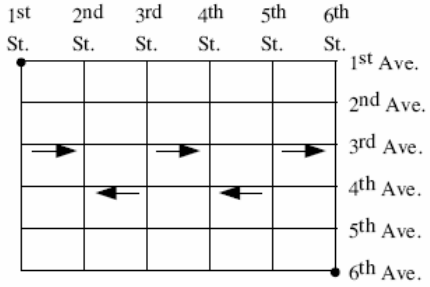
C. $\frac{8!}{4!\,4!} + \frac{6!}{3!\,3!}$ **D.** $\frac{8!}{4!\,4!} \times \frac{6!}{3!\,3!}$



Numerical Response **6.** The number of different paths that a pinball can take as it falls from top to bottom is ___ .



7. Counsellor McAdam has to drive from city hall (at the corner of 1st Ave. and 1st St.) to the hospital (at the corner of 6th Ave. and 6th St.). Third Avenue is one way heading east and Fourth Avenue is one way heading west. If he must always be travelling closer to the hospital, the number of different routes he can take is _____ .



Answer Key

1. **a)** 126 **b)** 60 **c)** 106 **d)** 140 **e)** 90
f) 900 **g)** 260 **h)** 495 **i)** 59 **j)** 66 **k)** 23

2. 35 **3.** 11 440. Assumptions: 1) He always travels closer to his destination.
2) None of the routes are one-way from north to south or east to west

4. **a)** Exit A: 1 Exit B: 5 Exit C: 10 Exit D: 10 Exit E: 5 Exit F: 1
b) Exit A: 20 Exit B: 15 Exit C: 6

5. B **6.** 53 **7.** 126