Probability Lesson 1:
Probability Terminology and Notation

Warm-Up

The following simple probability questions were asked in Junior High Math.

a) A fair die is rolled. What is the probability of rolling a "one" ?

b) A circular spinner is divided into four equal sectors, labelled clubs, diamonds, hearts and spades. When it is spun what is the probability it lands on "hearts"?

c) Two coins are thrown and the number of heads is counted. What is the probability of obtaining "two heads"?

Do not calculate the probabilities at this time.

Terminology

Probability theory deals with the mathematics of **chance** or **prediction**.
The following terminology will be used:

- A **trial** is any operation whose outcome cannot be predicted with certainty.
  eg. a coin is tossed
- An **experiment** consists of one or more trials.
  eg. a coin is tossed, three dice are rolled.
- An **outcome** is the result of carrying out an experiment
  eg. H, 6 6 4,
- The **sample space** (S) of an experiment is the set of all possible outcomes.
  eg. {H, T} , {1, 2, 3, 4, 5, 6} , {(1,1),(1,2), . . . (6,6)}
- An **event** is a subset of the sample space. It consists of one or more of the possible outcomes of an experiment.
  eg. {H} , {an even number}
- If an experiment has a set of **equally likely** possible outcomes then the probability of a particular event A is given by the formula;

$$P(A) = \frac{\text{number of outcomes favourable to } A}{\text{total number of possible outcomes}} \qquad \text{eg. } P(H) = \frac{1}{2}$$

- If the event X does not include any of the outcomes in the sample space, then the event X is **impossible** and we write $P(X) = 0$.
  eg. P(rolling a 9 on a standard die) = 0
- If the event Y includes all of the outcomes in the sample space, then the event Y is **certain** and $P(Y) = 1$.
  eg. P(rolling a natural number less than 7 on a standard die) = 1

- For any event A, $0 \le P(A) \le 1$.

- The notation $P(\overline{A})$ means the probability of "not A," the **complement** of A,

  i.e. $P(\text{not } A) = P(\overline{A}) = 1 - P(A)$

  eg. if $P(\text{rain tomorrow}) = \frac{1}{3}$, then $P(\textbf{no} \text{ rain tomorrow}) = 1 - \frac{1}{3} = \frac{2}{3}$

  The formula $P(\overline{A}) = 1 - P(A)$ is on the formula sheet.

a) For each of the examples in the *Warm Up* state:

i) the sample space
ii) the outcomes in the sample space which are favourable to the event indicated
iii) whether the outcomes are equally likely
iv) the probability of the event

- Warm-Up #1a)
  A fair die is rolled. What is the probability of rolling a "one" ?

  i) {1, 2, 3, 4, 5, 6} ii) {1} iii) Yes - Because each outcome occurs an equal number of times

  iv) $P(1) = \frac{1}{6}$

- Warm-Up #1b):
  A circular spinner is divided into four equal sectors, labelled clubs, diamonds, hearts and spades. When it is spun what is the probability it lands on "hearts"?

  i) {clubs, diamonds, hearts, spades} ii) {hearts} iii) Yes - because each outcome occurs an equal number of times.

  iv) $P(\text{hearts}) = \frac{1}{4}$

- Warm-Up #1c):
  Two coins are thrown and the number of heads is counted. What is the probability of obtaining "two heads"?

  i) {0 head, 1 head, 2 heads}
  TT TH, HT HH
  ii) {2 heads} iii) Not equally likely - because obtaining 1 head occurs twice as many times as obtaining 0 or 2 heads

  Toss 1 Toss 2
  H < H HH
  H < T HT
  T < H TH
  T < T TT
  4 outcomes

  iv) $P(HH) = \frac{1}{4}$

b) Why can you not directly use the formula $P(A) = \dfrac{\text{number of outcomes favourable to } A}{\text{total number of possible outcomes}}$ to calculate the probability in Warm-Up #1c)?

Because the outcomes are not equally likely - ie. Although the sample space shows a total of 3 different outcomes, the tree diagram shows 4 outcomes - one which appears twice combining for 3 different outcomes.

c) Explain how you calculated the probability.

Using the tree diagram, count up the favorable outcomes (one) and divide by the total number of outcomes.

$P(2H) = \frac{1}{4}$

Compound Events

Events formed from repeated trials or from a combination of simple events are called **compound events** and often a table, a chart or a tree diagram is useful in determining the sample space.

Class Ex. #2

Consider an experiment of rolling an equally spaced triangular spinner numbered 1 to 3 and tossing two coins.

a) Draw a tree diagram to show all the outcomes for the experiment.
List the sample space at the end of the tree.

b) How many elements are in the sample space? How can you use the fundamental counting principle to determine the answer?

• 12

$(3)(2)(2) = 12$

c) Are all the outcomes equally likely?

Yes

d) State the probability of obtaining :

i) a three and two heads $P(3HH) = \frac{1}{12}$

ii) a prime number and exactly one tail $P(\text{prime}, \text{one T}) = \frac{4}{12} = \frac{1}{3}$

Class Ex. #3

A blue die and a red die are rolled. The outcome "3 on the blue die and 4 on the red die" can be represented by the ordered pair (3, 4).

a) Show all the possible outcomes in the array.

b) How many points are in the sample space?

$6 \times 6 = 36$

c) List the event "the same number appears on both dice" as a subset of the sample space.

$\{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

Red Die / Blue Die

Blue Die \ Red Die	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3, 4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

d) State the probabilities of the following events:

i) the same number appears on both dice

$P(\text{doubles}) = \frac{6}{36} = \frac{1}{6}$

ii) a different number appears on each die

$P(\overline{\text{doubles}}) = 1 - P(\text{doubles}) = 1 - \frac{1}{6} = \frac{5}{6}$

Complete Assignment Questions #1 - #11

Assignment

1. A machine produces batteries. As part of a quality control process, six batteries are randomly selected every hour and tested. The number of defective batteries is noted.

a) List the sample space.

b) List the following events as subsets of the sample space:
i) at least two batteries are defective **ii)** at most two batteries are defective

c) List the complement of the following events as subsets of the sample space:
i) three batteries are defective **ii)** at most five batteries are defective

2. Eight coins are tossed and the number of tails is counted.

a) List the sample space.

b) List the event "more tails than heads" in terms of the sample space.

c) List the event "more heads than tails" in terms of the sample space.

d) List the complement of the following events as subsets of the sample space:
i) an even number of tails **ii)** at least one tail

3. A committee of two people is to be selected from;
Alex (A), Bob(B), Carol(C), Dave(D), and Ed(E).

a) List the sample space.

b) List the following events as subsets of the sample space:
i) Alex is on the committee **ii)** neither Carol nor Dave is on the committee

c) Assuming each element of the sample space to be equally likely, determine
i) P(Alex is on the committee) **ii)** P(Bob and Dave are on the committee)

iii) P(Alex is not on the committee)

4. Packet 1 contains an orange candy, a lemon candy and a strawberry candy.
Packet 2 contains an orange candy, a strawberry candy and a raspberry candy.
Without looking in the packets a student takes one candy from packet 1, then one candy from packet 2.

a) Use a tree diagram to determine the sample space for this experiment?

b) List the event "both candies are the same type" as a subset of the sample space.

c) List the event "a strawberry candy is not chosen" as a subset of the sample space.

d) State the probability of the following events:

i) both candies are the same

ii) a strawberry candy is not chosen

iii) at least one orange candy is chosen

iv) the candies are different types

e) Explain the relationship between the answers to parts **i)** and **iv)**

5. Four dimes are tossed by Student 1 and five quarters are tossed by Student 2.
The number of heads is recorded by each student.

a) List the sample space as an array of ordered pairs

b) List the event "Student 1 recorded more heads than Student 2" in terms of the sample space.

c) State the probability of the following events:

i) Student 1 recorded **more** heads than Student 2.

ii) Student 1 recorded **less** heads than Student 2.

d) Is the event in c)ii) the complement of the event in c)i)? Explain.

6. Rebecca, Elizabeth, and Jenny each toss a coin to see who is going to pay for coffee.

- If one of the students throws a different outcome from the other two then that student buys all three coffees.
- If all the students throw the same outcome then each student pays for their own coffee.

a) Use a tree diagram to construct a sample space for tossing three coins.

b) State the probabilities of the following events :

i) Rebecca buys all the coffees.

ii) each student buys their own

iii) Jenny gets a free coffee.

Multiple Choice

7. An experiment consists of rolling a die, flipping a coin, and spinning a spinner divided into 5 regions. The number of elements in the sample space of this experiment is

A. 3
B. 7
C. 13
D. 60

8. If $P(X) = 0.2$, then $P(\overline{X})$ equals

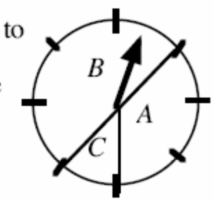
A. –0.2
B. 0.2
C. 0.5
D. 0.8

Numerical Response

9. Box A contains a red marble, a blue marble and a yellow marble. Box B contains a red marble, a blue marble, a yellow marble and a green marble.
One marble is taken from each box.
The probability, to the nearest hundredth, that both marbles are the same colour is _____ .

Use the following information to answer questions #10 - #11

Samantha spins a pointer at a carnival similar to the one shown. If the pointer lands on *A* she will win a medium sized stuffed rabbit. If the pointer lands on *B* she will a small stuffed teddy bear. If the pointer lands on *C* she will win a large stuffed unicorn.



10. The probability, to three decimal places, that Samantha wins a stuffed rabbit is _____ .

11. The probability, to the nearest hundredth, that Samantha does not win a unicorn is _____ .

Answer Key

1. a) {0, 1, 2, 3, 4, 5, 6} **b) i)** {2, 3, 4, 5, 6} **ii)** {0, 1, 2} **c) i)** {0, 1, 2, 4, 5, 6} **ii)** {6}

2. a) {0, 1, 2, 3, 4, 5, 6, 7, 8} **b)** {5, 6, 7, 8} **c)** {0, 1, 2, 3} **d) i)** {1, 3, 5, 7} **ii)** {0}

3. a) {AB, AC, AD, AE, BC, BD, BE, CD, CE, DE}
b) i) {AB, AC, AD, AE} **ii)** {AB, AE, BE} **c) i)** $\frac{2}{5}$ **ii)** $\frac{1}{10}$ **iii)** $\frac{3}{5}$

4. a) {OO, OS, OR, LO, LS, LR, SO, SS, SR} **b)** {OO, SS} **c)** {OO, OR, LO, LR}
d) i) $\frac{2}{9}$ **ii)** $\frac{4}{9}$ **iii)** $\frac{5}{9}$ **iv)** $\frac{7}{9}$
e) Event iv is the complement of event one so the probablilty in iv is 1 – probability in i)

5. a) 30 pairs from (0, 0) to (4, 5) **b)** {(1,0), (2,0), (3,0), (4,0), (2,1), (3,1), (4,1), (3,2), (4,2), (4,3)}
c) i) $\frac{1}{3}$ **ii)** $\frac{1}{2}$ **d)** No, because there are outcomes where # heads tossed by each student is the same

6. a) {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} **b) i)** $\frac{1}{4}$ **ii)** $\frac{1}{4}$ **iii)** $\frac{1}{2}$

7. D **8.** D **9.** 0.25 **10.** 0.375 **11.** 0.875