Probability Lesson #2:
Mutually Exclusive Events and the Event "A or B"

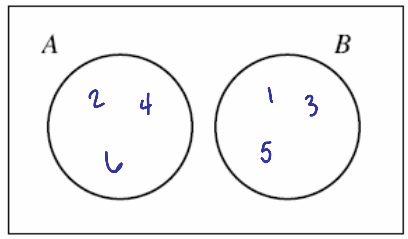
The Events "A or B", "A and B"

In mathematics the event A **or** B is said to occur if the event A occurs or if the event B occurs or if both events occur.

The event A **and** B occurs if both event A and event B occur simultaneously.

Warm-Up #1

Consider the experiment of rolling a die and noting the result. Let the event A be "an even number is thrown" and the event B be "an odd number is thrown".



a) Mark the outcomes to the experiment on the Venn Diagram which represents the sample space.

b) List the outcomes for:

i) the event A

$\{2, 4, 6\}$

ii) the event B

$\{1, 3, 5\}$

iii) the event A **or** B

$\{1, 2, 3, 4, 5, 6\}$

iv) the event A **and** B

$\{\ \}$, $\emptyset$ null or empty set

c) Let $n(A)$ represent the number of outcomes in event A.

Complete the following:

$n(A) =$ 3 $\quad n(B) =$ 3 $\quad n(A \text{ or } B) =$ 6 $\quad n(A \text{ and } B) =$ 0

d) Determine the following probabilities:

$P(A) = \frac{3}{6} = \frac{1}{2} \quad P(B) = \frac{3}{6} = \frac{1}{2} \quad P(A \text{ or } B) = \frac{6}{6} = 1 \quad P(A \text{ and } B) = \frac{0}{6} = 0$

- In this experiment the events A, B have **no common outcomes**.
  The events A, B are called **mutually exclusive**.
- Notice that in the Venn Diagram the circle for A and the circle for B have no area of overlap.

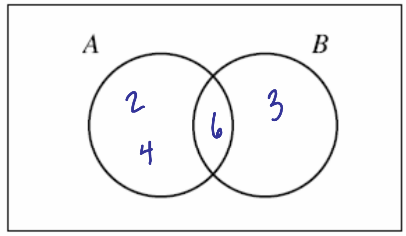
e) Verify the following rule.

$P(A \text{ or } B)$	$= P(A) + P(B)$
1	$= \frac{1}{2} + \frac{1}{2}$
1	$= 1$ ✓

Warm-Up #2

Consider the experiment of rolling a die and noting the result. Let the event A be "an even number is thrown" and the event B be "a multiple of three" is thrown.

a) Mark the outcomes to the experiment on the Venn Diagram.



b) List the outcomes for:

i) the event A {2,4,6}

ii) the event B {3,6}

iii) the event A **or** B {2,3,4,6}

iv) the event A **and** B {6}

c) Complete the following:

$n(A) =$ 3 $n(B) =$ 2 $n(A \text{ or } B) =$ 4 $n(A \text{ and } B) =$ 1

d) Determine the following probabilities:

$P(A) = \frac{3}{4}$ $P(B) = \frac{2}{4} = \frac{1}{2}$ $P(A \text{ or } B) = \frac{4}{4} = 1$ $P(A \text{ and } B) = \frac{1}{4}$

- In this experiment the events A, B **have** common outcomes. The events A, B are **not mutually exclusive**.

- Notice that in the Venn Diagram the circle for A and the circle for B have an area of overlap representing the event A and B.

e) Verify the following rule.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$1 = \frac{3}{4} + \frac{2}{4} - \frac{1}{4}$

$1 = 1$

Mutually Exclusive Events

Events are said to be **mutually exclusive** if they have no common outcomes.

Probability of the Event A or B

Use the following formulas for the probability of the event A **or** B.

If the events A, B, are **mutually exclusive** then

$$P(A \text{ or } B) = P(A) + P(B)$$

This formula is NOT on the formula sheet

If the events A, B, are **NOT mutually exclusive**, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

This formula is on the formula sheet

Class Ex. #1

In each case, state whether the events A, B are mutually exclusive or not.

a) Experiment - a card is drawn from a standard deck

Event A - a face card is selected   Event B - a club is selected

Not mutually exclusive: face card could be a club

b) Experiment - two dice are thrown

Event A - the dice both show the same value   Event B - the total score is 11

1,1 , 2,2 , 3,3 , 4,4 , 5,5 , 6,6

Mutually exclusive: No doubles add to 11

Class Ex. #2

Consider the experiment of drawing a card from a standard deck.
The following events are defined:

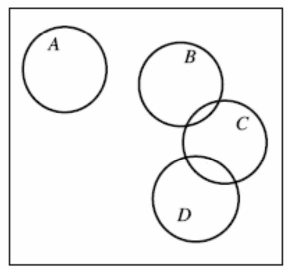
- Event A - a face card is selected
- Event B - a club is selected
- Event C - an ace is selected
- Event D - a red card is selected

State all the pairs of events which are mutually exclusive.

Mutually exclusive: events A and C , B and D (no common outcomes)

Class Ex. #3

Use the Venn Diagram to state all the pairs of events which are mutually exclusive.



Events AB , AC , AD , BD (no common outcomes) (no overlapp)

Class Ex. #4

Use the following information to determine whether the events A, B are mutually exclusive.

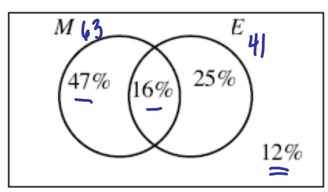
$$P(A) = \frac{1}{4} \qquad P(B) = \frac{1}{3} \qquad P(A \text{ or } B) = \frac{7}{12}$$

$P(A \text{ or } B)$	$P(A)+P(B)$
$\frac{7}{12}$	$\frac{1}{4}+\frac{1}{3}$
$\frac{7}{12}$	$\frac{3}{12}+\frac{4}{12}$
$\frac{7}{12}$	$\frac{7}{12}$ ✓

Mutually exclusive

Class Ex. #5

A grade 9 class were surveyed to find out whether they did Math homework or English homework last night. The Venn diagram shows the percentage of students in each category.



If a student is selected at random from the class determine the probability that the student last night did :

a) Math homework   **b)** Math and English homework   **c)** Math or English homework

$P(M) = 47 + 16 = 63\%$ $P(M \text{ and } E) = 16\%$ $P(\text{Math or English}) = 63 + 41 - 16 = 88$ or $1 - P(\overline{M \text{ or } E})$ $100 - 12 = 88$

Class Ex. #6

A card is drawn from a standard deck of 52 cards. Use formulas to determine the probability that:

a) A nine of diamonds or a heart is drawn.

$P(9\diamondsuit \text{ or } \heartsuit) = P(9\diamondsuit) + P(\heartsuit)$

$= \frac{1}{52} + \frac{13}{52} = \frac{14}{52}$

$= \frac{7}{26}$

no common outcomes so mutually exclusive

b) A nine or a heart is drawn. common outcomes

$P(9 \text{ or } \heartsuit) = P(9) + P(\heartsuit) - P(9 \text{ and } \heartsuit)$ Not mutually exclusive

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

$= \frac{16}{52} = \frac{4}{13}$

Class Ex. #7

200 people with neurology symptoms, which include headaches and backaches, participated in a study to evaluate a pain relief medicine. All the people took the medicine and the results were as follows:

- 60 people experienced headache relief
- 126 people experienced backache relief
- 36 people experienced relief from both

What is the probability that a person who takes the drug experiences relief from

a) at least one of the two symptoms?

$P(\text{at least one}) = P(H) \text{ or } P(B)$

$= P(H) + P(B) - P(H \text{ and } B)$

$= \frac{60}{200} + \frac{126}{200} - \frac{36}{200} = \frac{150}{200} = \frac{3}{4}$

b) neither of the symptoms?

$P(\text{neither}) = 1 - P(\text{at least one})$

$= 1 - \frac{150}{200}$

$= 1 - \frac{3}{4} = \frac{1}{4}$

Complete Assignment Questions #1 - #12

Assignment

1. In each case, state whether the events A, B are mutually exclusive or not.

a) Experiment - a card is drawn from a standard deck
Event A - a spade is selected   Event B - a club is selected

b) Experiment - a card is drawn from a standard deck
Event A - a spade is selected   Event B - a black seven is selected

c) Experiment - a card is drawn from a standard deck
Event A - a red two is selected   Event B - a red face card is selected

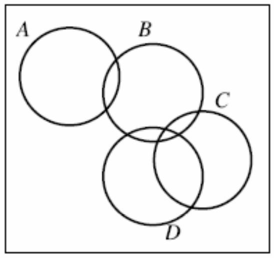
d) Experiment - two dice are thrown
Event A - the dice both show odd numbers   Event B - the total score is 8

e) Experiment - a chocolate bar is chosen
Event A - the bar contains raisins   Event B - the bar contains nuts

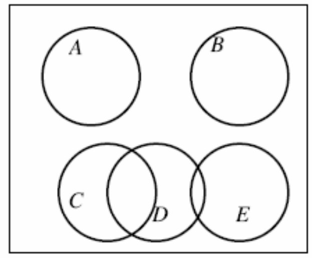
f) Experiment - a student is chosen from this class
Event A - the student is left handed   Event B - the student is female

2. Use the Venn Diagram to state all the pairs of events which are mutually exclusive.

a)



b)



3. Meghan (M), Jolene (J), and Tara (T) are the only entrants in a competition for which there is a first and second prize. Consider the experiment of selecting the two prize winners.

a) List the sample space.

b) State as subsets of the sample space the events

i) P – Meghan wins first prize **ii)** Q – Meghan wins second prize

iii) R – Meghan wins a prize **iv)** S – Jolene and Tara win the prizes

c) Which three of these events are mutually exclusive?

d) List all pairs of these events that are mutually exclusive.

4. Use the following information to determine whether the events A, B are mutually exclusive.

a) $P(A) = \frac{1}{2} \quad P(B) = \frac{1}{5} \quad P(A \text{ and } B) = 0$ **b)** $P(A) = 0.3, P(B) = 0.2, \ P(A \text{ or } B) = 0.4$

c) $P(A) = \frac{2}{5}, \ P(B) = \frac{1}{4}, \ P(A \text{ or } B) = \frac{13}{20}$

5. Use the given information to calculate the probability of the indicated event.

a) Tanya and Fujiko are two of the competitors in the final of the long jump competition. The probability that Tanya wins the competition is $\frac{2}{7}$, and the probability that Fujiko wins is $\frac{1}{5}$. Calculate the probability that Tanya or Fujiko win the competition.

b) In a group of students, the probability that a student chosen at random walks to school is 0.35, and the probability that a student has blonde hair is 0.2. If the probability that a student walks to school or has blonde hair is 0.45, calculate the probability that a student walks to school and has brown hair.

c) Mohinder is playing with an incomplete deck of cards. When one card is seclected at random from the incomplete deck, the probability that it is a heart is 0.3, and the probablity that it is a face card is 0.2. If the probability that it is a heart and a face card is 0.1, calculate the probability that it is a heart or face card.

6. A card is drawn from a standard deck. Use an appropriate formula to determine the probability that;

a) a face card or a club is drawn

b) a red card or a club is drawn.

c) an eight or a spade is drawn

d) a five of diamonds or a Queen is drawn

e) a black card or a diamond is drawn

f) a Jack or a red card is drawn.

7. Allyson and Brittney were solving the following problem:

100 people in a community participated in a technology survey. It was found that 80 people have a computer, 40 people have access to the internet, and 30 people have both a computer and access to the internet. If one of these people is chosen at random, what is the probability that the person has

a) *a computer or access to the internet?* ***b)*** *neither?*

Allyson chose to do the problem using a Venn diagram. Brittney chose to use probability formulas.

Show each student's method of solution below:

Allyson's Solution	**Brittney's Solution**

8. Mr. Spark has problems starting his two cars during the cold winter months. He decided to record the number of times each car starts during a one month period in the winter. He attempted to start each car every morning for thirty days. He recorded the following information.

- His first car started 20 times
- His second car started 18 times
- Both cars started 40% of the time

What is the probability that on any particular morning during the month

a) at least one of the cars start? **b)** he cannot start either of his two cars?

9. In a group of 80 elementary school children, 48 children liked cornflakes, 22 children liked both cornflakes and oatmeal, and 20 children liked neither cornflakes nor oatmeal. Calculate the probability that a child selected at random from the group liked only oatmeal.

10. In a school, 55% of the students have passed their English examination and 45% have passed their Mathematics examination. Comment critically on the following statement.

" The probability the student chosen at random has passed his or her English examination or Mathematics examination is $0.55 + 0.45 = 1$."

Use the following information to answer questions #11 and #12.

In an election there are three candidates. Candidate A is twice as likely to win as candidate B and candidate B is twice as likely to win as candidate C.

Multiple Choice

11. The probability that candidate A wins the election is

A. $\frac{2}{3}$ **B.** $\frac{3}{4}$ **C.** $\frac{4}{5}$ **D.** $\frac{4}{7}$

Numerical Response

12. The probability, expressed as a decimal to the nearest hundredth, that candidate A or candidate B wins the election is _____ .

Answer Key

1. **a)** yes **b)** no **c)** yes **d)** no **e)** no **f)** probably no (depends on the class)

2. **a)** A and C, A and D **b)** A and B, A and C, A and D, A and E, B and C, B and D, B and E, C and E

3. **a)** $\{MJ, MT, JM, JT, TM, TJ\}$

b) **i)** $\{MJ, MT\}$ **ii)** $\{JM, TM\}$ **iii)** $\{MJ, MT, JM, TM\}$ **iv)** $\{JT, TJ\}$

c) P, Q, and S **d)** P and Q, P and S, Q and S, R and S.

4. **a)** Yes, since $P(A \text{ and } B) = 0$ **b)** No, since $P(A) + P(B) \neq P(A \text{ or } B)$

c) Yes, since $P(A) + P(B) = P(A \text{ or } B)$ **5.** **a)** $\frac{17}{35}$ **b)** 0.1 **c)** 0.4

6. **a)** $\frac{11}{26}$ **b)** $\frac{3}{4}$ **c)** $\frac{4}{13}$ **d)** $\frac{5}{52}$ **e)** $\frac{3}{4}$ **f)** $\frac{7}{13}$

7. **a)** $\frac{9}{10}$ **b)** $\frac{1}{10}$ **8.** **a)** $\frac{13}{15}$ **b)** $\frac{2}{15}$ **9.** $\frac{3}{20}$

10. The two events are not mutually exclusive, so the statement is false **11.** D **12.** 0.86