Probability Lesson #4:
Probability Problems Involving Independent Events

Warm-Up #1 *Review of Probability Formulas*

If the events A, B, are **mutually exclusive**, then	If the events A, B, are **NOT mutually exclusive**, then
$P(A \text{ or } B) = P(A) + P(B)$	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

If the events A, B, are **independent**, then	If the events A, B, are **dependent**, then
$P(A \text{ and } B) = P(A) \times P(B)$	$P(A \text{ and } B) = P(A) \times P(B \mid A)$

Class Ex. #1

Two hockey players, Wayne and Mario, each independently take a penalty shot. Wayne has a $\frac{7}{10}$ chance of scoring, and Mario has a $\frac{3}{5}$ chance of scoring. What is the probability that;

$W = \frac{7}{10}$, $\overline{W} = \frac{3}{10}$, $M = \frac{3}{5}$, $\overline{M} = \frac{2}{5}$

a) both score? $P(W \text{ and } M) = P(W) \times P(M)$
$\frac{7}{10} \times \frac{3}{5}$
$= \frac{21}{50}$

b) both miss? $P(\overline{W} \text{ and } \overline{M}) = P(\overline{W}) \times P(\overline{M})$
$= \frac{3}{10} \times \frac{2}{5}$
$= \frac{6}{50} = \frac{3}{25}$

c) only one of them scores?
$P(W) \times P(\overline{M})$ or $P(\overline{W}) \times P(M)$
$[\frac{7}{10} \times \frac{2}{5}] + [\frac{3}{10} \times \frac{3}{5}]$
$\frac{14}{50} + \frac{9}{50} = \frac{23}{50}$

d) at least one of them scores?
$1 - P(\overline{W} \text{ and } \overline{M})$
$1 - \frac{3}{25} = \frac{22}{25}$

Class Ex. #2

In the World Cup Soccer final, the score at the end of regular time was Brazil 2 Italy 2. After five penalty kicks for each team, the game was still tied. The game will now be decided by a penalty shootout where each team takes alternate shots on goal. Each team shoots once in each round. If both teams score, or both teams miss, they go on to another round. From past records the probability that Brazil will score on a penalty shot is 0.7 and the probability that Italy will score on a penalty shot is 0.6. What is the probability that;

$P(B) = .7$, $P(\overline{B}) = .3$, $P(I) = .6$, $P(\overline{I}) = .4$

a) Brazil wins in the first round
$P(B \text{ and } \overline{I}) = .7 \times .4$
$= .28$

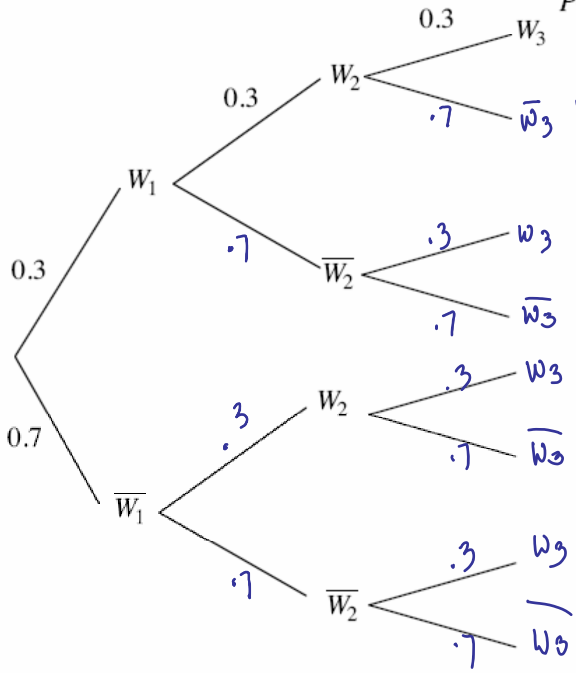
b) Italy wins in the first round?
$P(I \text{ and } \overline{B}) = .6 \times .3$
$= .18$

c) Brazil wins in the second round?
$[P(B \text{ and } I) \text{ or } P(\overline{B} \text{ and } \overline{I})] \text{ and } P(B \text{ and } \overline{I})$
$[(.7)(.6) + (.3)(.4)] \times (.7)(.4)$
$.42 + .12$
$[.54] \times [.28] = .1512$

Using a Probability Tree Diagram for Independent Events

Class Ex. #3

Johann is planning a trip during the forthcoming three-day holiday weekend. He is considering cancelling the trip if the weather will be wet. The probability of a wet day is 0.3 and the weather on each day is independent of the weather on the other days.



$P(W_1 \text{ and } W_2 \text{ and } W_3) = (.3)(.3)(.3) = 0.027$

$P(W_1 \text{ and } W_2 \text{ and } \overline{W_3}) = (.3)(.3)(.7) = 0.063$

$P(W_1 \text{ and } \overline{W_2} \text{ and } W_3) = (.3)(.7)(.3) = 0.063$

$P(W_1 \text{ and } \overline{W_2} \text{ and } \overline{W_3}) = (.3)(.7)(.7) = 0.147$

$P(\overline{W_1} \text{ and } W_2 \text{ and } W_3) = (.7)(.3)(.3) = 0.063$

$P(\overline{W_1} \text{ and } W_2 \text{ and } \overline{W_3}) = (.7)(.3)(.7) = 0.147$

$P(\overline{W_1} \text{ and } \overline{W_2} \text{ and } W_3) = (.7)(.7)(.3) = 0.147$

$P(\overline{W_1} \text{ and } \overline{W_2} \text{ and } \overline{W_3}) = (.7)(.7)(.7) = 0.343$

Complete the tree diagram to determine the probability that;

a) all 3 days will be wet

$P(W_1 \text{ and } W_2 \text{ and } W_3) = (.3)(.3)(.3) = 0.027$

b) there will be only 1 wet day

$P(W_1 \overline{W_2}\, \overline{W_3}) \text{ or } P(\overline{W_1} W_2 \overline{W_3}) \text{ or } P(\overline{W_1}\, \overline{W_2} W_3)$

$(.3)(.7)(.7) + (.7)(.3)(.7) + (.7)(.7)(.3)$

$.147 + .147 + .147 = .441$

c) that at least 2 days will be dry

$P(W_1 \overline{W_2}\, \overline{W_3}) \text{ or } P(\overline{W_1} W_2 \overline{W_3}) \text{ or } P(\overline{W_1}\, \overline{W_2} W_3) \text{ or } P(\overline{W_1}\, \overline{W_2}\, \overline{W_3})$

$.147 + .147 + .147 + .343 = .784$

Probability Problems Involving Infinite Geometric Sequences

Consider the following problem.

" Two friends Albert and Breanne are playing Scrabble. To decide who starts, they throw a die alternately until one of them throws a six. Albert throws first."

Find the probability that Albert starts the game of Scrabble.

Let A_i be the event that Albert throws a six on his i^{th} attempt.
B_i be the event that Breanne throws a six on her i^{th} attempt.

Albert will start the game of Scrabble if any one of the following events occurs:

Event	*Probability Notation*	*Probability*
Albert throws a six on his first attempt	$P(A_1)$	$\frac{1}{6}$
or		
neither Albert nor Breanne throws a six on their first attempt and Albert throws a six on his second attempt	$P(\overline{A}_1 \text{ and } \overline{B}_1 \text{ and } A_2)$	$\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \left(\frac{5}{6}\right)^2 \times \frac{1}{6}$
or		
neither Albert nor Breanne throws a six on their first two attempts and Albert throws a six on his third attempt	$P(\overline{A}_1 \,\&\, \overline{B}_1 \,\&\, \overline{A}_2 \,\&\, \overline{B}_2 \,\&\, A_3)$	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \left(\frac{5}{6}\right)^4 \times \frac{1}{6}$
or	$P(\overline{A}_1 \,\&\, \overline{B}_1 \,\&\, \overline{A}_2 \,\&\, \overline{B}_2 \,\&\, \overline{A}_3 \,\&\, \overline{B}_3 \,\&\, A_4)$	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \left(\frac{5}{6}\right)^6 \times \frac{1}{6}$

The probabilities in the extreme right column form an infinite geometric sequence.

a) Complete the next row of the table and use the results to determine the common ratio of the infinite geometric sequence.

$a = \frac{1}{6}$ $\quad r = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$

$$r = \frac{t_2}{t_1} = \frac{\left(\frac{5}{6}\right)^2 \times \frac{1}{6}}{\frac{1}{6}} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

b) Find the sum of the terms of this infinite geometric sequence to determine the probablilty that Albert starts the game.

$$S = \frac{a}{1-r} = \frac{\frac{1}{6}}{1-\frac{25}{36}} = \frac{\frac{1}{6}}{\frac{11}{36}} = \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

Complete Assignment Questions #1 - #10

Assignment

1. Alan and Steve each attempt a three-point shot in basketball. The probability that Alan is successful is 0.4, and the probability that Steve is successful is 0.35 and the events are independent. Calculate the probability that:

a) both players are successful

b) only one player is successful

c) at most one player misses.

2. Mike and Phil are playing in the final of a matchplay golf tournament. On the first hole, they each have a ten foot putt for birdie. The probability that Mike makes the putt is 0.6 and the probability that Phil makes the putt is 0.5, and these events are independent. Assuming that neither player will take more than two putts to complete the hole, calculate the probability that:

a) they both make the ten foot putt

b) Mike wins the hole

c) the hole is halved

3. In a Pee Wee hockey tournament a shootout consists of the Rebels and the Crusaders taking alternate shots on goal. Each team shoots once in each round. If both teams score, or both teams miss, they go on to another round. From past records the probability that the Rebels will score on a shot is 0.3 and the probability that the Crusaders will score on a shot is 0.2. What is the probability that;

a) Rebels win in the first round?

b) Crusaders win in the first round?

c) Rebels win in the second round?

4. Chess is a game played by two players. A game results in one of three mutually exclusive outcomes - either player A wins, or player B wins, or the game results in a draw. In a world championship chess match, the Russian champion Kozlov and the American champion Armstrong are involved in a sudden death playoff. In the playoff, Kozlov and Armstrong keep playing games until one of them wins a game. The first player to win a game wins the championship. From previous results, the probability that Kozlov wins a game is $\frac{1}{3}$ and the probability that Armstrong wins a game is $\frac{1}{4}$.

a) Explain why the probability that the first game is a draw is $\frac{5}{12}$.

b) Calculate the probability that the championship has not been decided after the second game.

c) Calculate the probability that Armstrong wins the championship within two games.

5. The probability that Andy solves a particular problem is $\frac{1}{3}$, that Barry solves that problem is $\frac{1}{2}$ and that Curtis solves the problem is $\frac{3}{5}$. Given these probabilities are independent, find the probability that the problem is solved by;

a) all three students **b)** none of the students **c)** only one of the students

6. The probability of being caught in a traffic jam in the morning rush hour is 0.6 on any particular week day. If the occurrence of traffic jams on different days are assumed to be independent of each other, find;

a) the probability that the journey is free from traffic jams for three consecutive week days.

b) the probability that there is a traffic jam on at least two out of three consecutive week days.

7. A student has difficulty in getting up for school, and so, to waken himself, he sets three alarm clocks to ring at the same time, as the noise from at least two alarms is necessary to waken him. Each alarm goes off independently. The probabilities that each alarm rings are 0.7, 0.8, and 0.9, respectively. Find the probabilities that:

a) all three alarms ring **b)** no alarm rings **c)** the student is awakened

8. Of the patients registered at a clinic, 40% attend Dr. Adams, 30% attend Dr. Barber, and the remainder attend Dr. Chang. For the next two patients, calculate the probabilities that;

a) they are both for Dr. Chang **b)** they are both for the same doctor.

c) at least one is for Dr. Barber.

Assume that the choice of doctor is independent from patient to patient.

9. The school table tennis champion plays two opponents A and B in turn until one of them defeats him. The probability that the champion beats A is 0.8, and the probability that the champion beats B is 0.7 . If the first game is against A, find the probability that it is A who beats the champion.

10. Two hockey teams, the Spartans and the Burners, have a sudden death penalty shootout to decide who wins the game. The teams take penalty shots in turns. The first team to score wins. The probability that the Spartans score on any penalty shot is 0.3 and the probability that the Burners score on any penalty shot is 0.4 . If the Spartans take the first penalty shot, determine the probability that they win the game.

Answer Key

1. **a)** 0.14 **b)** 0.47 **c)** 0.61 **2.** **a)** 0.3 **b)** 0.3 **c)** 0.5

3. **a)** 0.24 **b)** 0.14 **c)** 0.1488

4. **a)** There are three mutually exclusive outcomes with a total probability of 1.
So the probability of a draw is $1 - \left(\frac{1}{3} + \frac{1}{4}\right) = \frac{5}{12}$ **b)** $\frac{25}{144}$ **c)** $\frac{17}{48}$

5. **a)** $\frac{1}{10}$ **b)** $\frac{2}{15}$ **c)** $\frac{2}{5}$

6. **a)** 0.064 **b)** 0.648 **7.** **a)** 0.504 **b)** 0.006 **c)** 0.902

8. **a)** 0.09 **b)** 0.34 **c)** 0.51 **9.** $\frac{5}{11}$ **10.** $\frac{15}{29}$