

Probability Lesson #6: Bayes' Law

Warm-Up

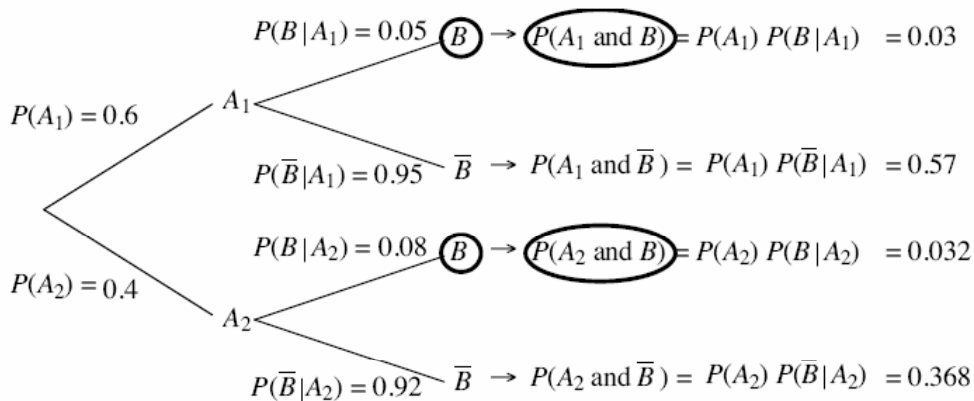
Developing Bayes' Law

Recall the following problem from Lesson #5:

“ Cheryl is trying to show Jon how to solve problems based on the following information.

“ Two machines A_1 and A_2 produce all the glass bottles made in a factory. Machine A_1 produces 60% of the output. The percentages of broken bottles produced by these machines are 5% and 8% respectively.” “

Jon set up the following probability tree diagram to calculate the probability of a broken bottle.



Jon used the formula $P(B) = P(A_1 \text{ and } B) + P(A_2 \text{ and } B)$ (circled above)
 $= P(A_1) P(B|A_1) + P(A_2) P(B|A_2) = 0.062$

Cheryl then posed the question.

“If a bottle selected at random is found to be defective, what is the probability it was produced by machine A_1 ?”

We are required to find $P(A_1|B)$.

This can be done using the conditional probability formula

$$P(A_1|B) = \frac{\text{1st branch}}{\text{1st branch} + \text{3rd branch}} = \frac{P(A_1 \text{ and } B)}{P(B)} = \frac{P(A_1) P(B|A_1)}{P(A_1) P(B|A_1) + P(A_2) P(B|A_2)}$$

Complete the work to determine the answer.

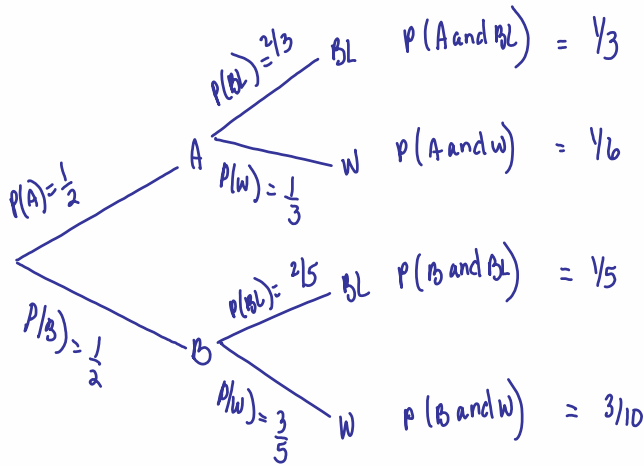
Bayes' Law

If a sample space is partitioned into mutually exclusive outcomes $A_1, A_2, A_3 \dots$ and if B is any other event then

$$P(A_1|B) = \frac{P(A_1 \text{ and } B)}{P(B)} = \frac{P(A_1) P(B|A_1)}{P(A_1) P(B|A_1) + P(A_2) P(B|A_2) + \dots}$$



Urn A contains 4 black balls and 2 white balls. Urn B contains 4 black balls and 6 white balls. An urn is selected at random and a ball is drawn from that urn. If the ball is black, what is the probability it came from urn A?

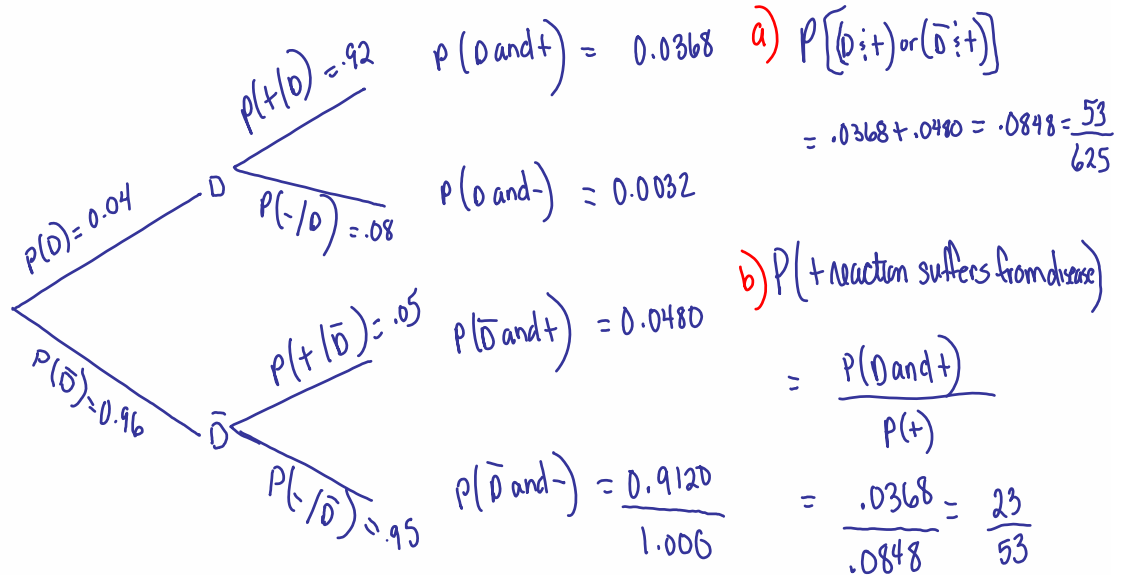


$$\begin{aligned}
 P(A|BL) &= \frac{P(A \text{ and } BL)}{P(BL)} \\
 &= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{5}} = \frac{\frac{1}{3}}{\frac{8}{15}} = \frac{5}{8}
 \end{aligned}$$



A new test for detecting a disease has been developed. Medical trials show that 92% of the patients who have the disease react positively to the new test, while 5% of patients not suffering from the disease also react positively. If 4% of the population have the disease, determine the probability that:

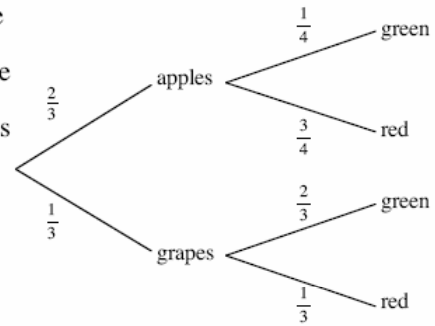
- a) a person reacts positively to the test.
- b) a person showing a positive reaction suffers from the disease.



Complete Assignment Questions #1 - #8

3. A school cafeteria sells small baskets of fruit. At the start of a school day, the only fruit available was green or red apples and green or red grapes. A grade 12 student constructed the tree diagram shown to represent the probabilities associated with the baskets of fruit.

The first student at the cafeteria bought one of the small baskets of fruit. Determine the following probabilities associated with her choice.



- a) $P(\text{apple})$ b) $P(\text{green apple})$ c) $P(\text{green}|\text{apple})$
- d) $P(\text{apple}|\text{green})$
4. There are two bus routes between Luke's home and downtown where he works. Luke uses route A two-thirds of the time, and route B one-third of the time. On route A, 10% of the buses are late, and on route B, 20% of the buses are late.
- a) What is the probability that Luke is late for work on a particular day?
- b) If Luke arrives late downtown, what is the probability he travelled by route A?

5. Box 1 contains 8 red dice and 4 blue dice. Box 2 contains 5 red dice and 7 blue dice. One card is randomly selected from a deck of playing cards. If the card is a diamond, a die is randomly selected from box 1. If the card is not a diamond, a die is randomly selected from box 2.
- Find the probability that the die selected is red.
 - Given that the die selected is red, calculate the probability it came from box 1.
-
6. A high school student can walk, drive, or take the bus to school. The number of school days for each method of transport are in the ratio 3:2:1. If the student walks to school, the probability she is late is 0.4. If she drives to school the probability she is late is 0.2 and if she takes the bus the probability she is late is 0.1.
- State the probability that on any particular day a student drives to school.
 - Construct a probability tree diagram and determine the probability that she is late on any particular day.

- c) If she was late today, calculate the probability that she travelled by:
- i) walking
 - ii) driving
 - iii) bus
7. A large group of university students take three examinations in the course of a semester. It is found that if a student passes an examination, the probability of his passing the next one is four-fifths, and if he fails an examination, the probability of his failing the next one is also four-fifths. In the first examination, 60% of the students pass.
- a) What is the probability that a student passes all three examinations?
 - b) If a student passes exactly two examinations, what is the probability that he failed the first one?

8. The following questions are extensions of questions in the assignment for Lesson #5. Use the tree diagrams from the assignments in Lesson #5 to answer the following.

a) on page 461, question #4

If the chosen balloon is red, calculate the probability it came from packet 2.

b) on page 461, question #5

If a person reacts positively to the test, what is the probability she actually has the disease?

c) on page 462, question #6

Given that the item is defective, calculate the probability it was produced by machine A.

Answer Key

1. $\frac{2}{3}$ 2. $\frac{24}{49}$ 3. a) $\frac{2}{3}$ b) $\frac{1}{6}$ c) $\frac{1}{4}$ d) $\frac{3}{7}$ 4. a) $\frac{2}{15}$ b) $\frac{1}{2}$
5. a) $\frac{23}{48}$ b) $\frac{8}{23}$ 6. a) $\frac{1}{3}$ b) $\frac{17}{60}$ c) i) $\frac{12}{17}$ ii) $\frac{4}{17}$ iii) $\frac{1}{17}$
7. a) $\frac{48}{125}$ b) $\frac{8}{23}$ 8. a) $\frac{5}{9}$ b) $\frac{2}{3}$ c) $\frac{25}{33}$