Probability Lesson #7:
Probability Involving Permutations and Combinations

Warm-Up *Using Permutations or Combinations to Find the Probability of an Event*

Two cards are picked without replacement from a deck of 52 playing cards. Determine the probability that both are kings using

a) the multiplication law

$P(K_1 \text{ and } K_2) = P(K_1) \cdot P(K_2 | K_1)$

$= \left(\frac{4}{52}\right)\left(\frac{3}{51}\right) = \frac{1}{221}$

b) combinations

$P(K_1 \text{ and } K_2) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}} = \frac{{}_4C_2}{{}_{52}C_2} = \frac{6}{1326} = \frac{1}{221}$

The word **COUNTED** has been spelled using Scrabble tiles. Two tiles are randomly chosen one at a time and placed in the order in which they were chosen. Determine the probability that the tiles are:

a) **CO** $P(\text{C and O}) = \frac{1}{7} \times \frac{1}{6} = \frac{1}{42}$

or $\frac{1}{{}_7P_2} = \frac{1}{42}$

b) both vowels $P(V_1 \text{ and } V_2) = \left(\frac{3}{7}\right)\left(\frac{2}{6}\right) = \frac{6}{42} = \frac{1}{7}$

or $\frac{{}_3P_2}{{}_7P_2} = \frac{6}{42} = \frac{1}{7}$

The Athletic Council decides to form a sub-committee of seven council members to look at how funds raised should be spent on sports activities at the school. There are a total of 15 athletic council members, 9 males and 6 females. What is the probability that the sub-committee will consist of exactly 3 females?

$P(3F) = \frac{\#\text{ ways to choose exactly 3 females}}{\#\text{ ways to choose a subcommittee}} = \frac{{}_6C_3 \cdot {}_9C_4}{{}_{15}C_7} = \frac{56}{143}$

A bag of marbles contains 5 red, 3 green, and 6 blue marbles. If a child grabs three marbles from the bag, determine the probability that:

a) exactly 2 are blue

$P(2B) = P(2B \text{ and 1other})$

$\frac{{}_6C_2 \cdot {}_8C_1}{{}_{14}C_3} = \frac{30}{91}$

b) at least one is blue

$P(1B + 2\text{other}) \text{ or } P(2B + 1\text{other}) \text{ or } P(3B + 0\text{others})$

$\frac{{}_6C_1 \cdot {}_8C_2 + {}_6C_2 \cdot {}_8C_1 + {}_6C_3 \cdot {}_8C_0}{{}_{14}C_3} = \frac{11}{13}$

c) the first is red, the second is green and the third is blue — Order Important

$P(1R \text{ and } 1G \text{ and } 1B)$

$\left(\frac{5}{14}\right)\left(\frac{3}{13}\right)\left(\frac{6}{12}\right) = \frac{15}{364}$

or $\frac{{}_5P_1 \cdot {}_3P_1 \cdot {}_6P_1}{{}_{14}P_3} = \frac{15}{364}$ ✓

d) one is red, one is green and one is blue — Order Not Important

$P(1R \text{ and } 1G \text{ and } 1B)$

$= 3! \times \left(\frac{5}{14}\right)\left(\frac{3}{13}\right)\left(\frac{6}{12}\right) = \frac{45}{182}$

$= \frac{{}_5C_1 \cdot {}_3C_1 \cdot {}_6C_1}{{}_{14}C_3} = \frac{45}{182}$ ✓

City Council consists of nine men and six women. Three representatives are chosen at random to form a environmental sub-committee.

a) What is the probability that Mayor Jim Milonovich and two women are chosen?

$P(\text{mayor and 2women}) = \dfrac{{}_1C_1 \cdot {}_6C_2 \cdot {}_8C_0}{{}_{15}C_3} = \dfrac{15}{455} = \dfrac{3}{91}$

b) What is the probability that two women are chosen if Mayor Jim Milonovich must be on the committee?

Mayor is already chosen

$P(\text{No men and 2women}) = \dfrac{{}_8C_0 \cdot {}_6C_2}{{}_{14}C_2} = \dfrac{15}{91}$

In a card game you are dealt 5 cards from a pack of 52 shuffled cards.
When you look at your 5 cards, what is the probability, expressed in combination notation, that you have:

a) four aces? $P(4A's \text{ \& } 1\text{other}) = \dfrac{{}_4C_4 \cdot {}_{48}C_1}{{}_{52}C_5} =$

b) four tens and an ace? $P(4, 10s \text{ and } 1A) \quad \dfrac{{}_4C_4 \cdot {}_4C_1}{{}_{52}C_5}$

c) 10, J, Q, K and ace? $\dfrac{({}_4C_1)^5}{{}_{52}C_5}$

d) at least one Jack? $1 - \text{No Jacks} \quad 1 - \dfrac{{}_4C_0 \cdot {}_{48}C_5}{{}_{52}C_5}$ or $\dfrac{{}_4C_1 \cdot {}_{48}C_4 + {}_4C_2 \cdot {}_{48}C_3 + {}_4C_3 \cdot {}_{48}C_2 + {}_4C_4 \cdot {}_{48}C_1}{{}_{52}C_5}$

In a class of 30 students, calculate the probability (to the nearest hundredth) that:

a) they all have different birthdays (assume no one is born on February 29)

$P(\text{all different Bdays}) = \dfrac{{}_{365}P_{30}}{365^{30}} = 0.29$

b) at least 2 of them have the same birthday.

Compliment = 1 − No same Bdays
$1 - .29 = 0.71$

Complete Assignment Questions #1 - #14

Assignment

1. A marble is drawn at random from a box containing 10 red, 30 yellow, 20 blue, and 10 pink marbles. Find, as an exact fraction, the probability that the marble drawn is:

a) yellow or red **b)** not blue **c)** green **d)** red, pink or blue

2. A bag contains 6 blue marbles and 10 yellow marbles. Two marbles are drawn from the bag without replacement.

a) Use the multiplication law to determine:

i) P(both are blue)

ii) P(the first is blue and the second is yellow)

iii) P(one is blue and one is yellow)

b) Use combinations or permutations to determine:

i) P(both are blue)

ii) P(the first is blue and the second is yellow)

iii) P(one is blue and one is yellow)

3. Two marbles are drawn at random from a box containing 10 red, 20 yellow, 15 blue and 15 pink marbles. Find the probability, to the nearest thousandth, that

a) both marbles are red **b)** the first marble is red and the second marble is yellow

c) neither marble is pink **d)** both marbles are either blue or red

4. In a card game you are dealt 5 cards from a pack of 52 well shuffled cards. What is the probability, expressed in combination notation, that you have:

a) four Queens? **b)** one face card? **c)** A, 2, 3, 4 and 5?

d) at most one King? **e)** at least one diamond?

5. Three prizes are rewarded in a raffle during a halftime show at a school basketball game. Ben, Janelle, Jamie, and 17 other students each have one ticket.

a) If the raffle has a first, second, and third prize, what is the probability, as an exact value, that Ben wins first prize, Janelle wins second prize, and Jamie wins third prize?

b) If the raffle has three identical prizes, what is the probability that Ben, Janelle, and Jamie win the prizes?

6. Three toys are selected at random from a box of nine toys, three of which are defective. Find the probability (exact value) that of the toys selected:

a) none are defective **b)** there are more defective than non-defective

7. A bank card personal identification number consists of any four digits. Repeat digits are allowed and the code can start with zero. What is the probability that a code begins and ends with 5?

8. In a small rural town there is only one football field to be shared by the five local teams. In the league competition, each team plays two games against every other team. What is the probability that the first game scheduled will have last year's two best teams, Oilmen and Wheatmen in opposition?

9. A jar contains 32 balls, equal numbers of red, blue, green and yellow. Five balls are drawn from the jar, simultaneously and at random. Determine the probability (to the nearest thousandth) of each event

a) There are three red balls and two yellow ones

b) The balls are three of one colour and two of another colour

c) All five balls have the same colour.

10. There are twelve boys and ten girls in an English 30 class. A yearbook committee of four is chosen at random.

a) What is the probability, to the nearest hundredth, that Ryan, the school president, and three girls are on the committee?

b) If Ryan must be on the committee, what is the probability, to the nearest hundredth, that three girls are chosen?

11. A six digit numeral is represented by the digits 1, 2, 3, 4, 5 and 6 in any order. If one of these numerals is chosen at random what is the probability that:

a) all the digits are in decreasing order? **b)** the even digits are in decreasing order?

Multiple Choice **12.** Chandra buys two of a total of thirty raffle tickets. There are two winning numbers. The probability that Chandra wins exactly one prize is

A. $\frac{56}{435}$ **B.** $\frac{28}{435}$ **C.** $\frac{2}{435}$ **D.** $\frac{1}{435}$

Numerical Response **13.** The letters of the word **OKOTOKS** are arranged. The probability, to the nearest hundredth, that an arrangement, chosen at random from all possible arrangements, begins and ends with a vowel is _____ .

14. The girls soccer team consists of 11 starters and 5 substitutes. The probability, to the nearest hundredth, that at least 2 of the 16 girls have the same birthday is _____ .

Answer Key

1. **a)** $\frac{4}{7}$ **b)** $\frac{5}{7}$ **c)** 0 **d)** $\frac{4}{7}$ **2.** **i)** $\frac{1}{8}$ **ii)** $\frac{1}{4}$ **iii)** $\frac{1}{2}$

3. **a)** 0.025 **b)** 0.056 **c)** 0.559 **d)** 0.169

4. **a)** $\frac{{}_4C_4\,{}_{48}C_1}{{}_{52}C_5}$ **b)** $\frac{{}_{12}C_1\,{}_{40}C_4}{{}_{52}C_5}$ **c)** $\frac{({}_4C_1)^5}{{}_{52}C_5}$ **d)** $\frac{{}_4C_0\,{}_{48}C_5}{{}_{52}C_5} + \frac{{}_4C_1\,{}_{48}C_4}{{}_{52}C_5}$ **e)** $1 - \frac{{}_{39}C_5}{{}_{52}C_5}$

5. **a)** $\frac{1}{6840}$ **b)** $\frac{1}{1140}$ **6.** **a)** $\frac{5}{21}$ **b)** $\frac{19}{84}$ **7.** $\frac{1}{100}$ **8.** $\frac{1}{10}$

9. **a)** 0.008 **b)** 0.093 **c)** 0.001

10. **a)** 0.02 **b)** 0.09

11. **a)** $\frac{1}{720}$ **b)** $\frac{1}{6}$ **12.** A **13.** 0.14 **14.** 0.28